

WAVE REFLECTION FROM A VERTICAL PERMEABLE WAVE ABSORBER

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ABSTRACT

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A theoretical solution for the reflection of linear shallow-water waves from a vertical porous wave absorber on a horizontal bottom is presented. Periodic solutions are matched at the front face of the absorber by assuming continuity of pressure and mass. The friction term describing the energy loss inside the absorber is linearized and, by using Lorentz principle of equivalent work, the reflection coefficient is determined as a function of parameters describing the incoming waves and the absorber characteristics.

INTRODUCTION

Prediction of the reflection from porous rubble mounds plays an important role in the assessment of the wave conditions in a harbour. A number of numerical short-wave models are able to account for this phenomenon but simple theoretical solutions are valuable when it comes to a quick investigation of the effect of varying the incoming waves or the rubble mound characteristics.

Theoretical solutions for the transmission of wave motions through permeable structures have previously been derived by several authors, for example Solitt and Cross (1972), Madsen (1974) and Madsen and White (1976). However, many breakwaters and piers are not homogeneous but consist of a number of layers with stone sizes decreasing towards the center of the structure, leading to an almost impermeable core. In this case the transmission through the rubble mound is eliminated but because of the energy dissipation inside the porous layers the reflection will only be partial. *This situation is similar to the case of wave absorbers applied for the damping of waves in laboratory experiments.*

In this paper a theoretical solution for the reflection of linear shallow-water waves from a vertical homogeneous wave absorber on a horizontal bottom is derived. Previous analytical approaches to the absorption problem

are made by Lean (1967) and by Svendsen (1976). However, Svendsen applied the long-wave approximations for the velocity outside as well as inside the absorber and hence did not satisfy the continuity equation. Lean, on the other hand, omitted the porosity in the matching of the velocities at the front face of the absorber and furthermore his theory was not really predictive as the resistance coefficient was not determined in terms of the incoming wave parameters and of the characteristics of the porous structure. Finally, a numerical model was presented by Nasser and McCorquodale (1975) describing the unsteady non-Darcy flow in a rockfill embankment with an impervious core. The method of characteristics and an explicit finite-difference technique were applied.

The solution technique used here will be similar to the one applied by Madsen and White (1976) for wave transmission through rubble mounds. An analytical expression for the reflection coefficient and an implicit expression for the friction factor (applying Lorentz' principle) will be given.

Actual breakwaters are often multilayered as well as of trapezoidal form and energy dissipation takes place not only inside the porous structure but also on the seaward slope due to friction. This leads to a two-dimensional problem where the vertical velocities must be taken into account. So far, no two-dimensional analytical solution has been derived, but approximative methods are suggested by Madsen and White (1976) for the case of transmitting rubble mounds. Numerical solution is possible, but has not yet been attempted.

ANALYTICAL SOLUTION FOR THE REFLECTION COEFFICIENT

The problem to be treated here is illustrated in Fig. 1. A rubble-mound structure with impermeable core is represented by an impervious wall with a porous structure in front of it. The structure is considered to be homogeneous and rectangular and the bottom to be horizontal. The incoming waves are assumed to be linear shallow-water waves which do not break at the entry to the absorber.

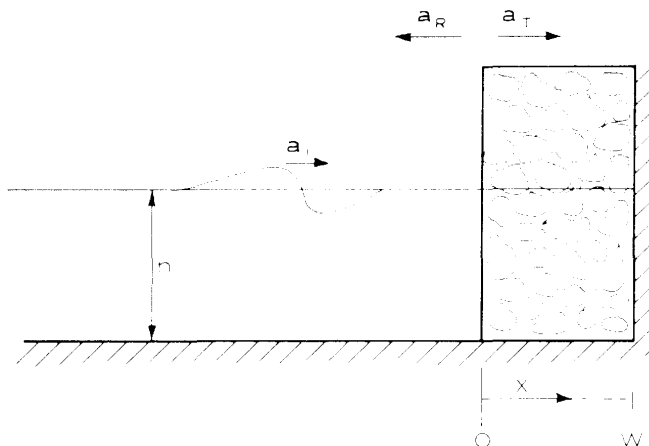


Fig. 1. Definition sketch.

The governing equations for the motion inside the absorber are:

$$n\dot{\zeta}_t + h U_x = 0 \quad (\text{continuity}) \quad (1a)$$

$$\frac{1}{n} U_t + g\dot{\zeta}_x + U(\alpha + \beta|U|) = 0 \quad (\text{momentum}) \quad (1b)$$

where α and β account for the laminar and turbulent friction loss respectively and n is the porosity of the structure.

Madsen and White (1976) included a coefficient attached to the acceleration term describing the effect of added mass. However, they concluded that the value assigned to this coefficient was of little consequence and that it could safely be taken as 1.

The solution technique used in the following will be similar to the one used by Madsen and White (1976) for wave transmission through porous structures. First of all the non-linear friction term is linearized by using the approximation

$$U(\alpha + \beta|U|) \cong f \frac{\omega}{n} U \quad (2)$$

where f is a friction factor which will be assumed to be independent of x and t .

Thus looking for periodic solutions of radian frequency, ω , we may express ζ and U in complex notations as:

$$\zeta = \text{Re}[\eta(x)e^{i\omega t}] , \quad U = \text{Re}[v(x)e^{i\omega t}] \quad (3)$$

By substituting eq. 3 into eq. 1 and eliminating U we find for η the equation:

$$\eta_{xx} + \frac{\omega^2}{gh} [1 - if]\eta = 0 \quad (4)$$

and for v :

$$v = -\frac{gn}{\omega} \frac{1}{f+i} \eta_x \quad (5)$$

Hence, according to Madsen and White (1976), the general solution for the flow within the porous structure is found to be:

$$\zeta = \text{Re}\{(a_1 e^{-i\kappa x} + a_2 e^{i\kappa x})e^{i\omega t}\}, \quad 0 \leq x \leq w \quad (6a)$$

$$U = \text{Re}\{(a_1 e^{-i\kappa x} - a_2 e^{i\kappa x})\sqrt{\frac{g}{h}} e^{i\omega t}\}, \quad 0 \leq x \leq w \quad (6b)$$

where

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$$\epsilon \equiv \frac{n}{\sqrt{1 - if}} \quad (7)$$

$$\kappa = \frac{\omega}{\sqrt{gh}} \sqrt{1 - if} \quad (8)$$

Svendsen (1976) found the same expression for ζ but did not use the continuity equation to obtain the velocity inside the absorber. The effect was equivalent to assuming that the long-wave approximations for the velocity could be applied inside as well as outside the permeable structure, corresponding to the omission of the ϵ coefficient in eq. 6b.

Outside the porous structure the linear shallow-water wave approximations lead to the solution:

$$\zeta = \text{Re} \{ a_i e^{i(\omega t - kx)} + a_r e^{i(\omega t + kx)} \}, \quad x \leq 0 \quad (9a)$$

$$U = \text{Re} \left\{ \sqrt{\frac{g}{h}} \left(a_i e^{i(\omega t - kx)} - a_r e^{i(\omega t + kx)} \right) \right\}, \quad x \leq 0 \quad (9b)$$

in which:

$$k = \omega / \sqrt{gh} \quad (10)$$

and a_i and a_r are the incoming and reflected wave amplitudes. At this stage the unknowns in the problem are the complex wave amplitudes a_1 , a_2 and a_r . These can be determined by applying the boundary conditions at the front face of the absorber and at the impervious wall.

Firstly a_2 can be eliminated by using the fact that the velocity has to be zero at the impervious wall ($x = w$). According to eq. 6b this leads to:

$$a_2 = a_1 e^{-i2\kappa w} \quad (11)$$

Secondly a_1 and a_r can be determined by assuming continuity of pressure (i.e. of surface elevation) and of mass (i.e. of velocity) at the front face of the absorber ($x = 0$). Hence according to eqs. 6, 9 and 11 this leads to:

$$a_i + a_r = a_1 (1 + e^{-i2\kappa w}) \quad (12a)$$

$$a_i - a_r = \epsilon a_1 (1 - e^{-i2\kappa w}) \quad (12b)$$

Solving eqs. 12a and 12b with respect to a_r yields:

$$\frac{a_r}{a_i} = \frac{1 - \epsilon + (1 + \epsilon)e^{-i2\kappa w}}{1 + \epsilon + (1 - \epsilon)e^{-i2\kappa w}} \quad (13)$$

According to eq. 9a the reflected wave can be expressed by:

$$\zeta_r = |a_r| \cos(\omega t + kx + \phi)$$

where a_r and ϕ are independent of x and t . Hence the reflection coefficient α_R is determined as the modulus of the complex reflection amplitude in eq. 13, i.e.

$$\alpha_R = |a_r|/a_i \quad (14)$$

This result yields α_R as a function of the friction factor (f), the porosity (n) and of the wavenumber multiplied by the width of the absorber (kw).

It can be shown that using ϵ equal to:

$$\frac{1}{\sqrt{1-if}} \text{ instead of } \frac{n}{\sqrt{1-if}}$$

the reflection coefficient becomes equal to the expression derived by Lean (1967). The reason for this discrepancy in ϵ is that Lean neglected the porosity in the matching of the velocities outside and inside the absorber.

The solution of α_R as a function of f , n and kw is very easily obtained by calculating the complex function (eq. 13) and its modulus on a computer. An example is shown on Fig. 2 where the reflection coefficient is drawn as a function of kw for a fixed value of f and for two different values of the porosity. The oscillating nature of the solution is seen to damp out for large values of kw corresponding to very long absorbers. On the other hand it is noticed that some reflection will occur no matter how long the absorber

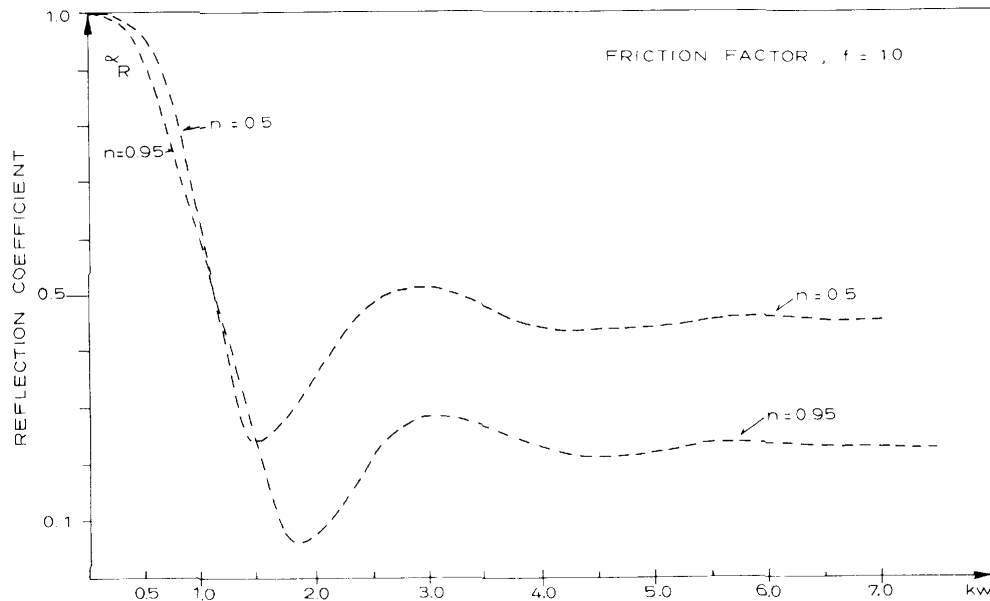


Fig. 2. Reflection from a porous wave absorber as a function of the wave number multiplied by the width of the absorber for fixed values of the friction factor and of the porosity.

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becomes:

For $kw \rightarrow \infty$, eq. 13 will simplify to:

$$\frac{a_r}{a_i} \rightarrow \frac{1 - \epsilon}{1 + \epsilon} \quad \text{for } kw \rightarrow \infty \quad (15)$$

As pointed out by Lean (1967), this situation corresponds to neglecting the reflective component (a_2) within the porous structure because the wave will be damped out before it reaches the impervious wall, from which it should be totally reflected.

It can easily be shown that neglecting a_2 in eqs. 6a and 6b leads to the result eq. 15.

For absorbers of very small width ($kw \ll 1$) the reflection becomes almost total. As seen from Fig. 2, the curve is very flat for small values of kw and it turns out that a Taylor expansion of the exponentials in eq. 13 would have to include terms of order $O(kw)^3$ to give anything different from unity. As a comparison Madsen and White (1976) needed only to include first-order terms in the case of transmission through rubble mounds of small width.

DETERMINATION OF THE FRICTION FACTOR

The reflection coefficient has been derived as a function of the friction factor (f), the porosity (n) and of the wavenumber multiplied by the width of the absorber (kw). However, in order to make a predictive solution, f which was formally introduced by eq. 2 must be related to parameters describing the incoming waves as well as the absorber characteristics. To do this the Lorentz' principle of equivalent work is applied. This principle states that the average rate of energy dissipation should be identical whether evaluated using the true non-linear resistance law or its linearized equivalent:

$$\int_0^W \int_0^T f \frac{\omega}{n} U^2 dt dx = \int_0^W \int_0^T (\alpha + \beta|U|) U^2 dt dx$$

in which T is the wave period and w the width of the absorber.

The value of U to be used in eq. 16 should correspond to the general solution for the flow inside the absorber. Solving eqs. 12a and 12b with respect to a_1 and substituting this result combined with eq. 11 into eq. 6b yields:

$$U = a_i \sqrt{\frac{g}{h}} \operatorname{Re} \left\{ \frac{2\epsilon(e^{-i\kappa x} - e^{i\kappa(x-2w)})}{1 + \epsilon + (1 - \epsilon)e^{-i2\kappa w}} e^{i\omega t} \right\}, \quad 0 \leq x \leq w \quad (17)$$

α and β in eq. 16 are determined by the empirical formulas by Englund

(1953):

$$\alpha = \alpha_0 \frac{(1-n)^3}{n^2} \frac{v}{d^2} \quad (18a)$$

$$\beta = \beta_0 \frac{(1-n)}{n^3} \frac{1}{d} \quad (18b)$$

in which d is the grain size, v the kinematic viscosity and α_0 and β_0 are particle form constants which in the following computations have been taken to be 1000 and 2.8, respectively.

Substituting eqs. 17, 18a and 18b into eq. 16 and rearranging terms finally leads to an equation which can be used for the determination of f :

$$F = 0 \quad (19)$$

where:

$$F \equiv \alpha_0 \frac{(1-n)^3}{n} \left(\frac{vT}{2\pi d^2} \right) + \beta_0 \frac{(1-n)}{n^2} \frac{a_i}{2\pi d} T \sqrt{\frac{g}{h}} \Lambda \cdot f \quad (20a)$$

$$\Lambda \equiv \frac{\int_0^W \int_0^T |U^*| U^{*2} dt dx}{\int_0^W \int_0^T U^{*2} dt dx} \quad (20b)$$

$$U^* = \text{Re} \left\{ \frac{2\epsilon(e^{-i\kappa x} - e^{i\kappa(x-2w)})}{1 + \epsilon + (1 - \epsilon)e^{-i2\kappa w}} e^{i\omega t} \right\} \quad (20c)$$

Notice that Λ depends on the friction factor so that eq. 19 has to be solved by numerical iteration.

An outline of the procedure is given below:

- (a) Assume two initial values for f , say $f = 0$ and $f = 1$.
- (b) Compute ϵ , κ and U^* for these two values.
- (c) Compute Λ by numerical integration and determine F for these two values of f .
- (d) Determine a new value of f by using the secant method:

$$f_{n+1} = \frac{F(f_n)f_{n-1} - F(f_{n-1})f_n}{F(f_n) - F(f_{n-1})}$$

- (e) Iterate if necessary.

For an accuracy requirement of

$$\frac{f_{\text{new}} - f_{\text{old}}}{f_{\text{old}}} = 1 \times 10^{-3}$$

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the iteration scheme typically closes sufficiently after 3–7 cycles.

Having computed the friction factor f , the reflection coefficient α_R can be determined from eq. 14 as a function of

- (a) the porosity, n ;
- (b) the width of the absorber, w ;
- (c) the grain size, d ;
- (d) the water depth, h ;
- (e) the wave period, T ;
- (f) the incoming wave amplitude, a_i .

Finally it should be remarked that in the simple case where the velocity inside the porous structure (U^*) is independent of x , the double integral in eq. 20b can be determined analytically to be:

$$\Lambda = \frac{8}{3\pi} |U^*|$$

This simplification was used by Madsen and White (1976) and by Lean (1967).

DISCUSSION OF THE RESULTS

Examples of the theoretical solution (eq. 13) are shown in Fig. 3–10. The reflection coefficient is shown as a function of the width of the absorber, the incoming wave height and of the porosity.

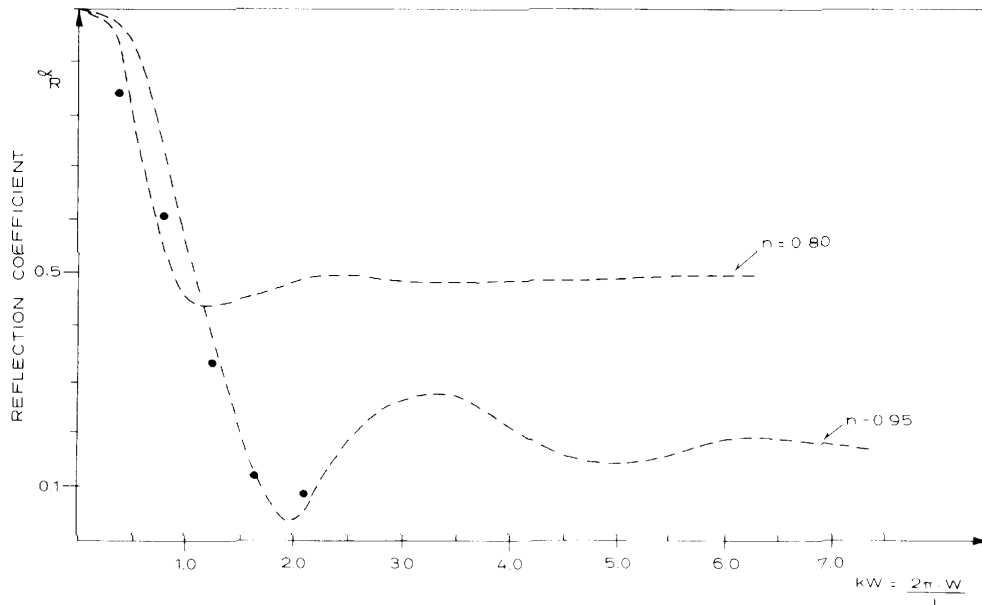


Fig. 3. The reflection coefficient as a function of the width of the absorber. --- = theoretical solution; • = numerical short wave model ($DT = 1$ sec, $DX = 15.5$ m, $n = 0.95$). Basic parameters: diameter of stones, $d = 0.2$ m; wave period, $T = 17.3$ sec; wave height, $H = 1.74$ m; water depth, $h = 21$ m; $\alpha_0 = 1000$; $\beta_0 = 2.8$.

The theory is compared with solutions obtained by a numerical short-wave model (Abbott et al., 1981) solving the vertically integrated Boussinesq equations. Porosity is included in these equations and the energy dissipation inside the permeable structure is represented by the non-linear term from eq. 16: $U(\alpha + \beta|U|)$. One-dimensional tests have been made with this numerical model and the reflection coefficients have been computed from line plots of the wave envelopes using Healy's formula,

$$\alpha_R = \frac{a_{\max} - a_{\min}}{a_{\max} + a_{\min}} \quad (21)$$

In Fig. 3 the reflection coefficient is shown as a function of the width of the absorber relative to the wave length. The curve is seen to be that of a damped oscillation with smaller and smaller oscillations for increasing values of kw . Because of the oscillations, it appears that a long absorber does not necessarily absorb more energy than a shorter one. On the other hand it appears that in order to be efficient, rectangular wave absorbers should be at least $\frac{1}{4}$ of the wave length. However, it is interesting to notice that some reflection will occur no matter how long the absorber. The limiting reflection coefficient (for $kw \rightarrow \infty$) was determined in eq. 15. It turns out that

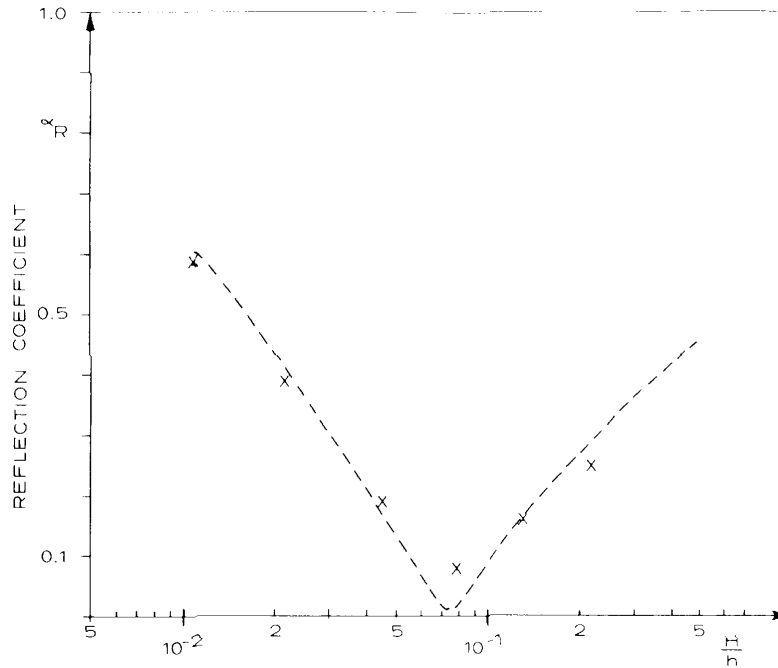


Fig. 4. The reflection coefficient as a function of the wave height. - - - = theoretical solution; \times = numerical short wave model ($DT = 1$ sec, $DX = 15.5$ m). Basic parameters: porosity, $n = 0.95$; diameter of stones, $d = 0.2$ m; width of the absorber, $w = 77.5$ m; wave period, $T = 17.3$ sec; water depth, $h = 21$ m.

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the agreement between the simplified theoretical solution and the numerical short wave model is entirely satisfactory (Fig. 3).

In Fig. 4 the reflection coefficient is shown as a function of the wave height. It turns out that this curve can be interpreted as a combination of the transmission and reflection from a rubble mound: the higher waves will be reflected from the front face of the absorber so that this part of the curve resembles the reflection curve from a rubble-mound (see for example Madsen and White, 1976). On the other hand, the lower waves will penetrate freely into the absorber where a double transmission will be performed because of the reflection from the impervious vertical wall at the end of the absorber. Hence, this part of the curve will resemble the transmission curve from a rubble-mound. For some intermediate wave height maximum absorption will occur. This will be the case when the wave height is small enough to allow for a reasonable amount of energy to be transmitted into the absorber and when the height is large enough to result in a significant dissipation of the transmitted energy inside the absorber.

The results obtained by the numerical short-wave model and the simplified theoretical solution are seen to be in excellent agreement (Fig. 4).

In Fig. 5 the theoretical solution is shown as a function of H/h for two different values of the porosity, 0.5 and 0.95. It is seen that efficient absorption can be obtained for both of the porosities, although the range of wave heights for which the two absorbers are optimal is completely different. For

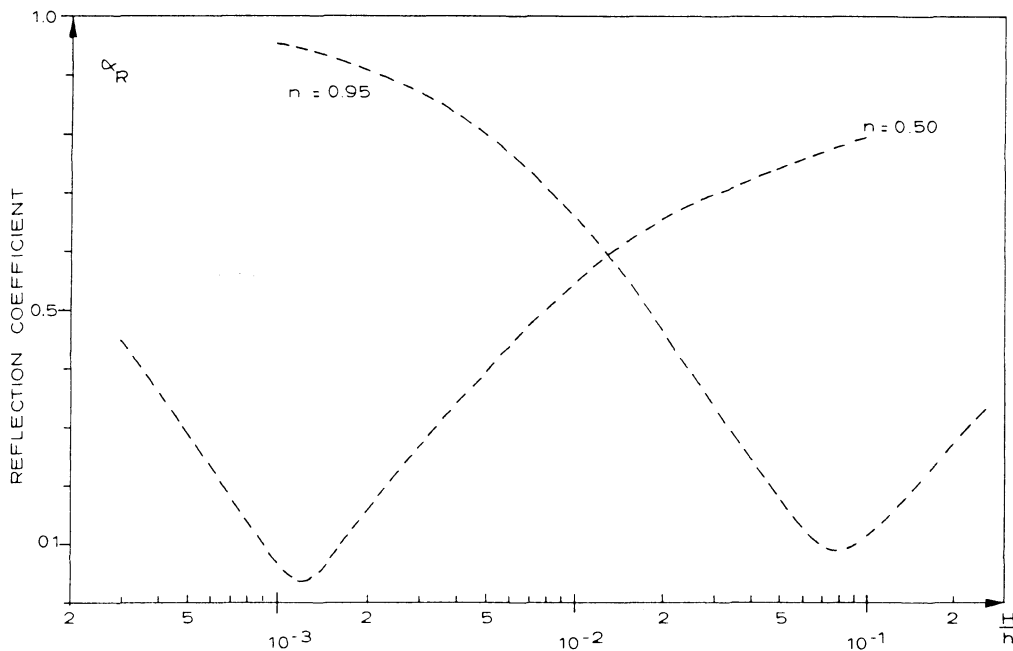
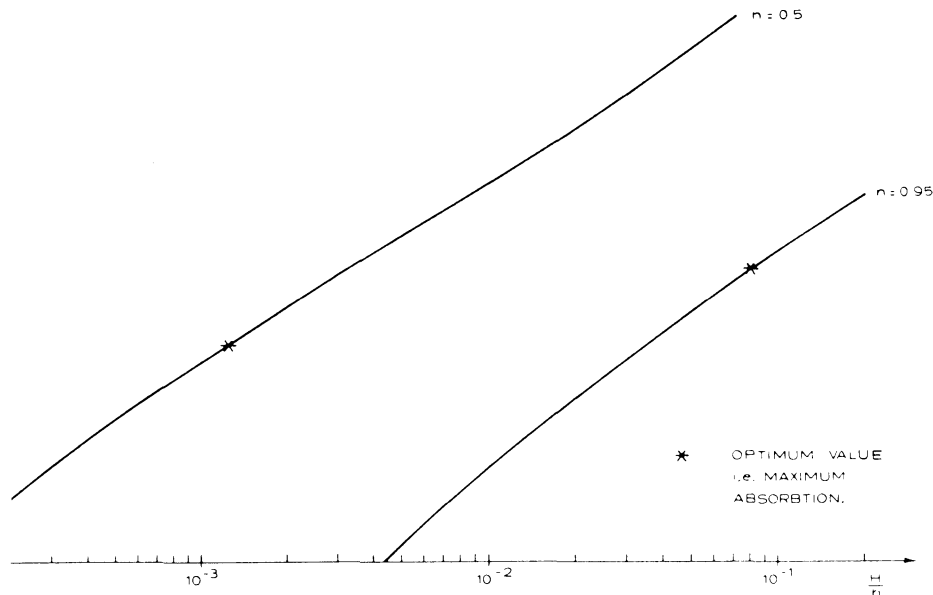


Fig. 5. The reflection coefficient as a function of H/h for two different porosities. Basic parameters: diameter of stones, $d = 0.2$ m; width of the absorber, $w = 77.5$ m; wave period, $T = 20$ sec; water depth, $h = 21$ m.



3. The friction factor as a function of H/h for two different porosities. Basic parameters: See Fig. 5.

higher waves a porosity of 0.95 is much more efficient than a porosity of 0.5. This is in qualitative agreement with the measurements by Straub (1966). In his experiments wire mesh plates were used to obtain porosities as high as 0.9 to 0.95. On the other hand the porosity 0.5 is optimal for absorbing very low waves, in which case the high porosity results in almost full absorption.

The efficiency of the absorber can be explained in terms of the friction factor, f . When f is large (low porosity combined with high waves) the absorber will act as an impermeable barrier and complete reflection ($\alpha_R = 1$) will occur at the front face. On the other hand, when f is zero (porosity equal to 1) no energy dissipation will occur and complete reflection is observed from the other end of the absorber. Thus there is an optimum value of f for which the reflection is a minimum. It turns out that the minimum reflection occurs for quite low values of f . This is illustrated by Fig. 6 which shows the friction factors corresponding to the solutions from Fig. 5. In both cases ($n = 0.5$ and 0.95) the optimal absorption is obtained for $f = 0.1$.

One remark should be made in connection with Figs. 5 and 6. As discussed by Solitt and Cross (1974), the description of the porous flow by the equations (1a) and (1b) is best when the incident wave height exceeds the particle diameter of the medium. As the wave amplitude becomes very small, the waves will begin to interact with individual pieces of breakwater material, leading to a partial reflection directly off the particle surfaces. In this case the theoretical assumption of a continuum no longer applies. However,

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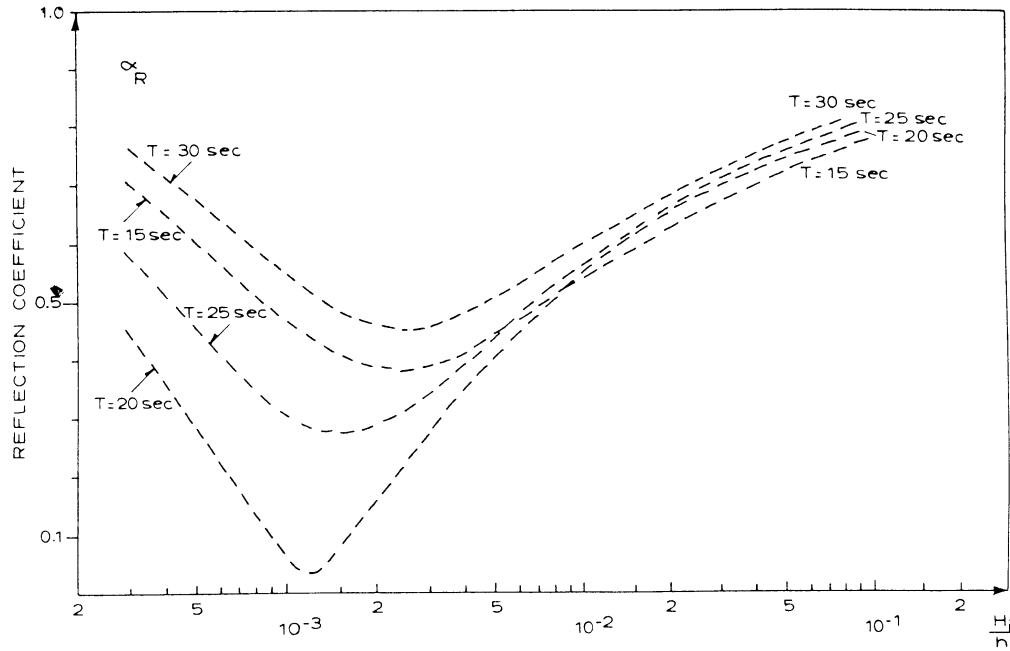


Fig. 7. The reflection coefficient as a function of H/h for a number of wave periods. Basic parameters: porosity, $n = 0.5$ m; width of the absorber, $w = 77.5$ m; diameter of stones, $d = 0.2$ m; water depth, $h = 21$ m.

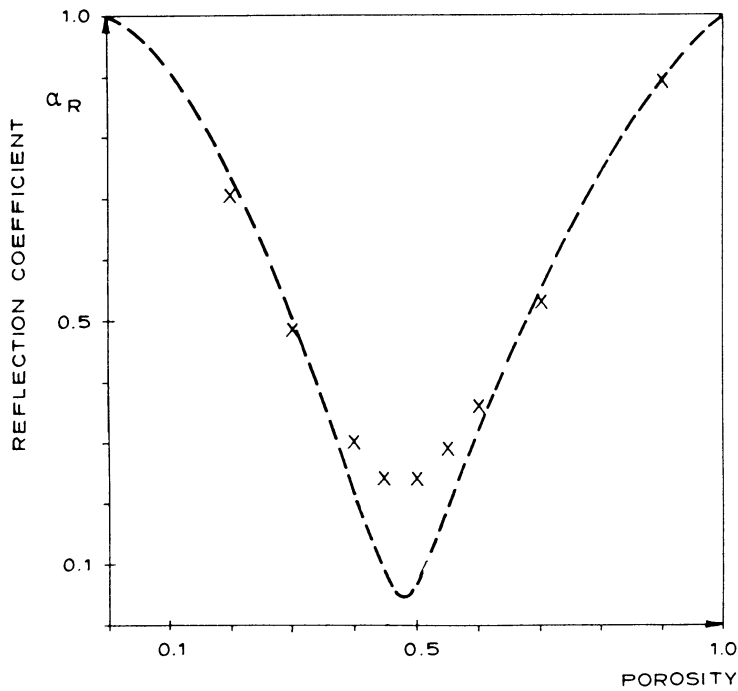


Fig. 8. The reflection coefficient as a function of the porosity. - - - = theoretical solution; x = numerical short wave model ($DT = 1$ sec, $DX = 15.5$ m). Basic parameters: diameter of stones, $d = 0.2$ m; width of the absorber, $w = 77.5$ m; wave period, $T = 20$ sec; water depth, $h = 21$ m; wave height, $H = 0.021$ m.

although one might question the correctness of the solution in Figs. 5 and 6 for the lowest waves, this does not change the conclusions concerning the differences in using a high or a low value of the porosity.

In Fig. 7 the reflection coefficient is shown as a function of the wave height for a number of wave periods. Notice that long waves are not necessarily more difficult to absorb than shorter waves: the absorption depends on the specific combination of the governing parameters.

In Figs. 8 and 9 the variation of the reflection coefficient with the porosity is shown for two different values of the incoming wave height. Once again it is noticed that high values of the porosity should be applied to absorb high waves while lower porosities are more efficient in case of lower wave heights. In Fig. 8 the theoretical solution as well as the numerical short wave model predict a minimum reflection to occur for a porosity of 0.47 but the value of this minimum reflection is different in the two cases. However, on the whole, the agreement is satisfactory.

In Fig. 9 the comparison leaves something to be desired. The discrepancies for high values of α_R are probably due to the way the reflection coefficient is determined from the wave field computed by the numerical short wave model. As explained earlier, Healy's formula (eq. 21) is applied and a_{\max} and

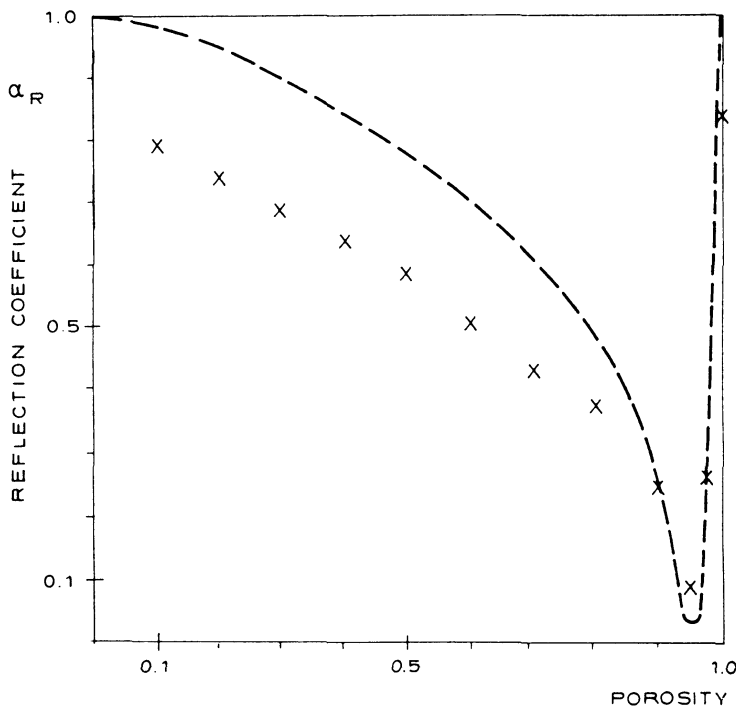


Fig. 9. The reflection coefficient as a function of the porosity. - - - = theoretical solution; X = numerical short wave model ($DT = 1$ sec, $DX = 15.5$ m). Basic parameters: diameter of stones, $d = 0.2$ m; width of the absorber, $w = 77.5$ m; wave period, $T = 17.3$ sec; water depth, $h = 21$ m; wave height, $H = 1.74$ m.

a_{\min} are determined from the wave envelope, which is in turn found by making a number of line plots of the surface elevation during a wave period. Now complete reflection, $\alpha_R = 1$, means a_{\min} equal to zero, but these kinds of nodal points will only occur if the waves are strictly linear. Non-linear effects will have a significant effect on a_{\min} and the resulting reflection coefficient determined by eq. 21 will be lower than the true reflection coefficient. Healy's formula is especially sensitive to non-linear effects for small values of a_{\min} , i.e. for large values of α_R . This is confirmed by Fig. 9 where a porosity of 1 leads to α_R equal to 0.84 instead of 1! Hence, although the discrepancies in Fig. 9 are due to non-linear effects, they are by no means a measure of the difference between reflection of linear and non-linear waves.

It has not been possible to locate any literature concerning experiments

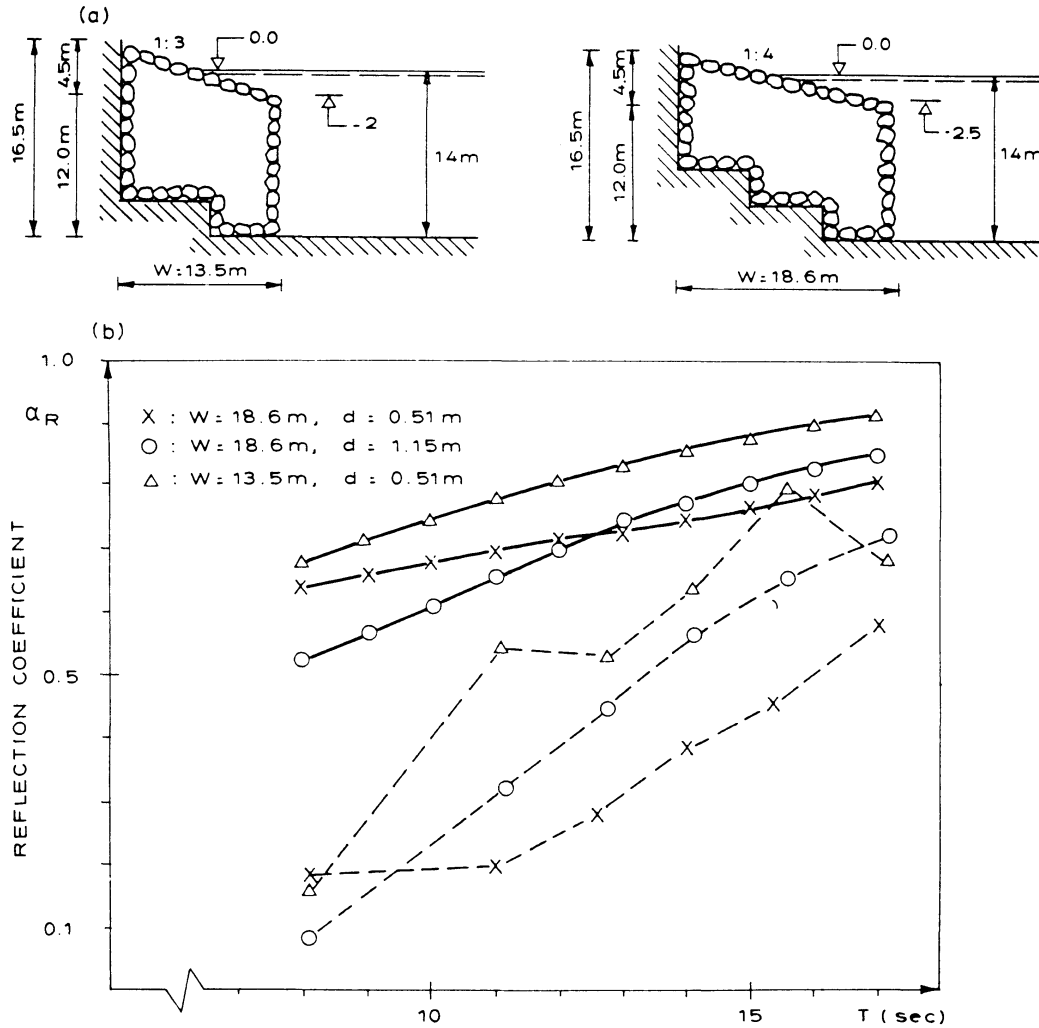


Fig. 10.a. Experimental setup. b. Comparison between theory and measurements. - - - = measurements (setup shown in Fig. 10a); — = theoretical solution. Basic parameters: water depth, $h = 14$ m; wave height, $H = 1$ m; porosity, $n = 0.4$.

with rectangular wave absorbers and although a lot of experimental data is reported by Straub (1956), it can only be used qualitatively for comparison purpose. A quantitative comparison has been made against measurements with almost-rectangular absorbers at the Danish Hydraulic Institute (Fig. 10a,b). However, it is seen that the surface slopes of these absorbers (1:3 to 1:4) have increased the efficiency considerably relative to the theoretical solutions predicted for rectangular absorbers (Fig. 10b). Relatively, the three theoretical reflection coefficients are similar to the corresponding three experimental results, but the absolute values of the theoretical solutions are much higher.

SUMMARY AND CONCLUSIONS

A theoretical solution for the reflection of linear shallow-water waves from a vertical-face homogeneous porous wave absorber has been derived. The bottom is assumed to be horizontal, the incident waves are supposed to be normal to the structure and wave breaking at the entry to the absorber is not considered.

The solution technique is similar to that used by Madsen and White (1976) for wave transmission through porous structures. First of all, the flow resistance in the porous material is linearized. This makes it possible to determine an analytical expression for the reflection coefficient as a function of the porosity, the friction factor and the wavenumber multiplied by the width of the absorber. Secondly, the friction factor is related to parameters describing the incoming waves and the absorber characteristics. This is done by employing Lorentz' principle of equivalent work and an implicit expression for f is obtained. This expression is solved by combined numerical integration and iteration and as a result the reflection coefficient is determined as a function of:

- (a) the porosity;
- (b) the width of the absorber;
- (c) the diameter of stones or grains;
- (d) the water depth;
- (e) the wave period;
- (f) the incoming wave height.

The theory has been compared against solutions obtained by a numerical short-wave model (Abbott et al., 1981) and the agreement is found to be most satisfactory. This means that the theoretical solution can be used in the calibration of numerical short wave models.

However, it should be emphasized that when considering reflection of irregular wave trains, the theoretical solution cannot be applied. This is due to the solution being strongly non-linear with respect to wave height as well as to wave length, which makes it impossible to apply the solution on the energy spectrum. In this case numerical short-wave models have to be used.

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