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A Sediment Transport Model for Straight Alluvial Channels

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The paper presents a simple mathematical model for sediment transport in straight alluvial channels. The model, which is based on physical ideas related to those introduced by Bagnold (1954), was originally developed in two steps, the first describing the bed load transport (Engelund 1975) and the second accounting for the suspended load (Fredsøe and Engelund 1976). The model is assumed to have two advantages as compared with empirical models, first it is based on a description of physical processes, secondly it gives some information about the quantity and size of the sand particles in suspension and the bed particles.

Introduction

One of the basic difficulties in sediment transport theory is the definition of bed load versus suspended load, and some authors have even tried to overcome the problem by disregarding any such distinction. However, there are several good reasons for maintaining it, mostly, of course, related to the nature of the physical processes. The authors would like to draw the attention to the fact that the instability of erodible beds which leads to the formation of sand dunes (or antidunes) can only be explained satisfactorily by a theory which clearly distinguishes between bed load and suspended load. It is found that transition between dunes and plane bed is very sensitive to a correct estimation of these transport rates (Engelund and Fredsøe 1974, Fredsøe and Engelund 1975, and Fredsøe 1976a). The stability analysis even gives an indication of one relevant way of defining the difference between the transport of bed load and suspension, respectively: The bed load is that part of the total load which accomodates to spatial changes in the tractive stress, so that spatial lag may be neglected, assuming inertia of the bed particles to be negligible. On the other hand, the suspended load responds with a certain lag, because the particles have to settle a certain distance before they become deposited. This lag depends on flow conditions and sediment properties and can be estimated from an equation of continuity.

Also in other problems in the field of river morphology and sedimentation a clear distinction between bed load and suspended load is important. For example it was found (Engelund 1976) that the transverse bed slope in river bends increases in linear proportion to the ratio q_s/q_B. Concerning the sedimentation of river navigation channels is, it was demonstrated (Fredsøe 1976b) that the rate of sedimentation for longitudinal currents is a function of the bed load, rather than of the total load. Further it might be mentioned that observations seem to indicate that the occurrence of meandering or braiding depends on the ratio q_s/q_B, so that the greater the relative amount of suspension is, the more pronounced is the tendency towards braiding. This tendency is until now not fully understood.

An obvious possibility is to define the bed load as the particles in the lowest layer of moving grains. Typically the particles move by rolling, sliding, or in short jumps. This definition is in accordance with the original definition by H. A. Einstein (1950): Bed load is the bed particles moving in the so-called »bed layer«, defined as »a flow layer, 2 grain diameters thick, immediately above the bed.«

In this treatise Einstein presented one of the first theoretical approaches to the problem of predicting theoretically the rate of bed load transport, applying theory of probability to account for the statistical variation of the forces acting on bed particles. If the magnitude of the instantaneous agitating forces on a certain bed particle exceeds the stabilizing forces, the particle is supposed to be eroded and to start moving along the bed, until it becomes deposited downstream at a location, where the magnitude of the instantaneous forces makes deposition possible. From such consideration Einstein found that the rate of bed load transport could be described by a relation between two non-dimensional quantities

$$\Phi = \frac{q_B}{\sqrt{(s-1)gd^3}} \quad \text{and} \quad \theta = \frac{\tau_0}{(s-1)\rho g d} = \frac{U_f^2}{(s-1)gd} \tag{1}$$

in which

q_B = rate of bed load transport in volume of material per unit time and unit width of the channel
s = relative density of sediment
g = acceleration of gravity
d = fall diameter of sediment particle
ρ = fluid density
τ_0 = bed shear stress (=tractive stress)

U_f = friction velocity $=\sqrt{\tau_0/\rho} = \sqrt{gDI}$
D = mean depth
I = energy gradient (slope)
Φ is a non-dimensional form of the transport rate, while
θ is the non-dimensional tractive stress (Shield's parameter).

When calculating the transport rate of the suspended load Einstein applied the concentration distribution

$$\frac{c}{c_a} = \left(\frac{D-y}{y} \quad \frac{a}{D-a}\right)^z \tag{2}$$

in which

c = concentration of suspended sediment (at y above the bed)
c_a = concentration at reference level ($y = a$)
D = depth of water
y = distance from bed level
$z = w/0.4\ U_f$, (the Rouse number) where w is the settling velocity.

Eq. (2) was derived by Ippen and Rouse (1937) and experimentally verified by Vanoni (1946). It suffers from the drawback that c_a usually cannot be predicted. The present paper suggests a method for calculation of c_a, based on a single dynamical principle. When c_a is known the transport rate q_s is found from

$$q_s = \int_a^D c\, U\, dy \tag{3}$$

U being the mean flow velocity at the distance y from the bed. Einstein's paper contains some excellent graphs which facilitate this calculation quite considerably.

In case the bed is covered by dunes the shear velocity Uf should be replaced by

$$U'_f = \sqrt{gD'I} \tag{4}$$

in which the reduced depth D' is found from the equation (Einstein 1950)

$$\frac{V}{U'_f} = \frac{V}{\sqrt{gD'I}} = 6 + 2.5 \ln \frac{D'}{k} \tag{5}$$

where k is the surface roughness, which is usually a little larger than the sediment size and may be taken as 2.5 d (Engelund and Hansen 1972). V is the mean velocity of the flow.

R. A. Bagnold (1954) pointed out a short-coming of the previous theories by formulating the following paradox: Consider the ideal case of fluid flow over a bed of uniform, perfectly piled spheres in a plane bed, so that all particles are equally exposed, statistical variations due to turbulence being neglected.

When a gradually increasing tractive stress exceeds a critical value, all particles in the upper layer are peeled off simultaneously and are dispersed in the fluid. Hence the next layer of particles is exposed to the flow and should consequently also be peeled off. The result is that all subsequent underlying layers are also eroded, so that a stable bed could not exist at all, when the shear stress exceeds the critical value.

Bagnold explained the paradox by assuming that in a water-sediment mixture the total shear stress τ would be separated in two parts

$$\tau = \tau_F + \tau_G ,$$

where τ_F is the shear stress transmitted by the intergranular fluid, while τ_G is the shear stress transmitted because of the interchange of momentum caused by the encounters of solid particles, i.e. tangential dispersive stress.

Hence, Bagnold's description of the physical process is, that when a layer of spheres is peeled off, some of the spheres may go into suspension while others will be transported as bed load. Thus a dispersive pressure on the next layer of spheres will develop and act as a stabilizing agency. Hence, a certain part of the total bed shear stress τ is transmitted as a grain shear stress τ_G and a correspondingly minor part as a fluid stress ($\tau_F = \tau - \tau_G$). Continuing this argumentation, it is understood that exactly so many layers of spheres will be eroded that the residual fluid stress τ_F on the first immovable layer is equal to the critical tractive stress τ_c. Hence, the mechanism in transmission of a tractive shear stress τ greater than τ_c is the following: τ_c is transferred directly by fluid shear stress to the immobile bed while the residual stress $\tau - \tau_c$ is transferred to the moving particles and further from these to the fixed bed as a dispersive stress.

By theoretical and experimental research Bagnold developed the following expression for the dispersive shear stress due to the grain collisions

$$F_S = 0.013 \; \rho s \, \lambda^2 d^2 \left(\frac{dU}{dy}\right)^2 \tag{6}$$

where λ is the so-called linear concentration, which is related to the volume concentration by the equation

$$c = \frac{0.65}{(1+1/\lambda)^3} \tag{7}$$

and y is the distance from the bed.

We shall later revert to an application of these expressions.

In recent publications (R. Fernandez Luque 1974, R. Fernandez Luque and R. van Beek 1976) certain modifications of Bagnold's ideas were suggested together with a consistent theory for the transport of bed load on a plane bed, considering the motion of individual particles. The theory is supported by a series of interesting experimental observations.

One of the basic issues is that close to incipient particle motion (i.e. small transport rates) only the topmost grains will be eroded, and the bed load will not effectively reduce the fluid part of the turbulent bed shear stress. This can also hardly be expected as, under these conditions, the bed load particles cover only a small portion of the bed surface, because only few particles will be in motion. According to the model of Fernandez Luque et al. the bed load particles reduce the maximum fluid shear stress at the bed surface to the critical value τ_c by exerting an average reaction force on the surrounding fluid. Hence the bed load forms a »protection shield« at higher bed load concentrations, which controls the erosion rate. We shall revert to this idea later on.

The experiments presented by these authors are of special interest, because they avoid the complication of dune influence, as the observations were carried out for very small transport rates mostly before bed waves became appreciable.

The Rate of Bed Load Transportation q_B

In the following it is attempted to approach the bed load transportation problem by considering the motion of the individual particles. Information about the transport velocity of single particles has been obtained by experiments published by Meland and Normann (1966). These experiments were carried out with single spherical glass beads moving over a bed of rhombohedrally packed spherical beads. In some of the runs the moving single particle was of the same size as those of the bed, while in other cases the bed particles were either larger or smaller. Following these authors notation d denotes the diameter of the migrating particle, while k is the diameter of the bed particles.

On the basis of these measurements it is attempted to develop a semi-empirical law for the mean transport velocity U_B of a particle moving as bed load. To this end we consider the most important forces determining the motion of an immersed particle:

1) The agitating forces, drag F_D and lift F_L, and
2) the stabilizing forces, reduced gravity (immersed weight) and the frictional forces resulting from the occasional contacts between particle and bed.

An exact description of forces and particle motion is impossible due to the complex character of the phenomenon. What we can do is to establish a »model equation« containing the time-averaged quantities, and from this equation we can hope to obtain sufficient information to be able to identify the parameters necessary for a relevant description of the process.

The agitating forces may be represented in form of a drag

$$c \, \frac{1}{2} \, \rho \, [\alpha U_f - U_B]^2 \, \frac{\pi}{4} \, d^2$$

in which U_f and U_B are the friction velocity and the migration velocity of the particle, respectively. αU_f is the flow velocity at a distance of about one or two grain diameter d from the fixed bed. Assuming the validity of the ordinary velocity distribution in rough channels, α must be of the order of 6 to 10. The factor c stands for the drag (and lift) coefficient, but as the time variation of the agitating forces differs considerably from that of the stabilizing forces, we can hardly expect the value of c to be exactly equal to the static value.

The frictional force acting on the particle is written as

$$\rho g(s-1)\frac{\pi}{6}d^3\beta$$

where s is the relative density of the particles, and β is the dynamic friction coefficient. Actually the gravity of the particle should be corrected for a small dynamic lift F_L. However, this may be done by changing the value of c.

The model equation then expresses the average equilibrium of agitating and stabilizing forces

$$c\,\frac{1}{2}\,\rho\,(\alpha U_f - U_B)^2\,\frac{\pi}{4}\,d^2 = \rho g(s-1)\,\frac{\pi}{6}\,d^3\,\beta\;, \tag{8}$$

from which

$$\frac{U_B}{U_f} = \alpha\,[1 - \sqrt{\theta_0/\theta}\,] \tag{9}$$

where θ is given by Eq. (1), and

$$\theta_0 = \frac{4\beta}{3\alpha^2 c} \tag{10}$$

θ_0 is seen to be the limiting value of θ for which a particle located on the bed is just immobile. It is natural to relate this to the critical value θ_c corresponding to Shield's criterion. As a particle lying on the bed is easier to move than a particle located in the bed, it must be expected that $\theta_0 < \theta_c$. A crude estimate can be obtained from Eq. (10) by insertion of the following values, partly obtained by the subsequent analysis:

$$\beta = \tan 27^0\,, \qquad \alpha = 9\,, \qquad c = 0.6\;,$$

which gives $\theta_0 = 0.014$, which is between one half and one fourth of the generally accepted values of θ_c.

It is probably better to evaluate θ_0 by considering the experiments of Fernandez Luque et al., which indicated θ_0 to be about half θ_c, so that Eq. (9) may be written

$$\frac{U_B}{U_f} = \alpha\,[1 - 0.7\,\sqrt{\theta_c/\theta}\,] \tag{11}$$

Comparison of this expression with Meland and Normann's result indicated that for suitable choice of θ_c and $\alpha = 10$, a very good agreement is obtained, as demonstrated in Fig. 1. Fernandez Luque's results, also indicated in this figure, are more satisfactory in the sense that θ_c was measured directly and U_B was determined as the mean transport velocity in a natural bed.

Eq. (11) was first suggested by Fernandez Luque and was checked by experiments with different slopes of the bed surface. It was found to hold irrespective of the inclination angle δ, provided the proper value of θ_c was inserted. The experiments indicate that we must take

$$\theta_c = \theta_{c,0}\,[1 + \tan\delta] \tag{12}$$

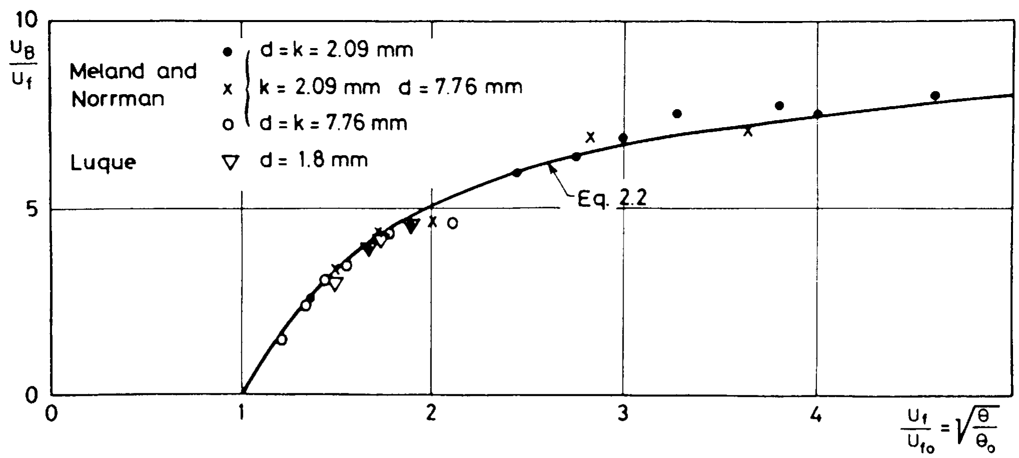

Fig. 1. Experiments of the transport velocity U_B of bed load particles.

where $\theta_{c,0}$ is the value of θ_c for horizontal bed. δ is positive when the particles move uphill.

For sand the value of α was found to be 9.3.

From the knowledge of mean particle velocity we can now derive an expression for the rate of bed load transport q_B under the assumption that the bed load is the transport of a certain fraction p (= probability) of the particles in a single layer. As the total number of surface particles per unit area is $1/d^2$, we get

$$q_B = \frac{\pi}{6} d^3 \frac{p}{d^2} U_B$$

or, after insertion of Eq. (11)

$$q_B = 9.3 \frac{\pi}{6} dp \; U_f \left[1 - 0.7 \sqrt{\theta_c/\theta}\,\right]$$

This is made non-dimensional by the divisor $\sqrt{(s-1)gd^3}$ (of Eq. (1)):

$$\Phi_B = 5p \left(\sqrt{\theta} - 0.7\sqrt{\theta_c}\right) \tag{13}$$

From the experiments of Fernandez Luque et al. we can get some empirical information about p, because the measurements comprise ϕ_B, θ, and θ_c. The result is given in Fig. 2, where the values of p calculated from Eq. (13) are plotted against θ. The experiments are particularly interesting because the transport rates were so small that all particles moved as bed load and without the disturbing effect of dunes and ripples. In case of larger transport rates this technique is no longer applicable, so that we must find other ways to obtain the necessary information.

Another useful series of data is that presented by Guy et al. (1966). For the present purpose, the evaluation of p, the experiments with sand size d = 0,93 mm are particularly suited as explained below. Because of the dune formations it is neccessary to divide the total shear stress in two components

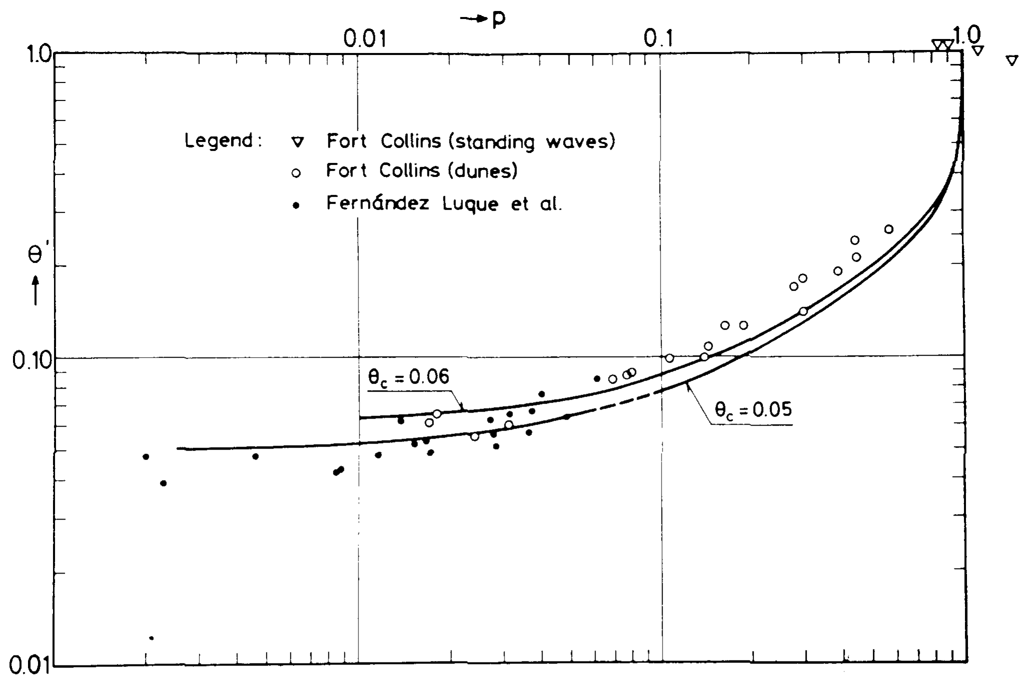

Fig. 2. The probability function p versus θ'.

$$\tau = \tau' + \tau'' \quad , \qquad \frac{\tau'}{\rho} = U_f'^2$$

where only the first term, which corresponds to the skin friction, is directly active in the bed load process. τ' (or U_f') is calculated from Einstein's procedure of Eqs. (4) and (5). Similarly, we have the non-dimensional version of Eq. (13).

$$\theta = \theta' + \theta''$$

In all the expressions derived above, θ should consequently be substituted by θ' in order to correct for the effect of dunes.

Another difficulty in applying ordinary flume data is that the total sediment load usually has a component of suspension, which has to be subtracted from the total load in order to obtain the bed load proper. The selected series of experiments has the particular advantage that the amount of suspended load was always a minor part of the total.

There are two different methods of evaluation of the bed load transport. The first one is to take the total load and subtract the measured suspended load. The second one is to consider the measured dune height h and the migration velocity a and then apply the equation

$$q_B \simeq \frac{1}{2}(1-m)\, a\, h \tag{14}$$

where m is the porosity of the sand bed.

Of course there is a considerable amount of uncertainty associated with both methods, which makes it so important to select experiments where the suspension is small. In the runs considered, the two methods were found to agree fairly well. Hence, after making an estimate of θ_c (e.g. by Shield's diagram), we can now calculate the value of p from Eq. (13), substituting the calculated value of θ' for θ. The result of such an analysis is also shown in Fig. 2, where the Fort Collins points and Fernandez Luque's data are seen to form a fairly consistent picture.

Now we are able to make a first check on the above-mentioned assumption that only the part τ_c of the total shear stress τ is transferred directly to the immobile bed as skin friction, while the residual part $\tau - \tau_c$ is carried as drag on the moving bed particles and indirectly transferred to the bed by occasional encounters. This idea leads to the equation

$$\tau \doteq \tau_c + n F_D \quad ,$$

where F_D is the average drag on a single moving bed particle, while n is the number of moving particles per unit area. If this expression is divided by $\rho g(s-1)d$, and F_D is estimated as

$$F_D \approx \rho g(s-1) \frac{\pi}{6} d^3 \beta$$

the resulting equation becomes

$$\theta = \theta_c + \frac{\pi}{6} \beta (nd^2) = \theta_c + \frac{\pi}{6} \beta p \qquad (15)$$

From this p can be determined, if θ_c and β are known. From investigations of flow in meanders (Engelund 1975, Gottlieb 1976) the value of the dynamic friction angle φ is known to be slightly smaller than the static (i.e. angle of repose), the value $\varphi = 27^0$ being reasonable for ordinary sand. Taking $\theta_c = 0.05$ we get

$$\theta = 0.05 + 0.2668\, p \quad ,$$

which is given in Fig. 2 for comparison with Fernandez Luque's experiments. The agreement is acceptable for small values of θ, but for the larger values the curves fall below the Fort Collins data. The explanation for this seems to be that Eq. (15) is only valid as long as there is no suspension, a point we shall revert to later in greater detail.

The next problem is what happens at very large transport rates. In this extreme the argument leading to Eq. (15) does obviously not hold. If we stick to the model that the bed load is a single layer of particles, the maximum value of p must be unity corresponding to a simultaneous motion of all particles in the layer.

In the Fort Collins series four runs (corresponding to »standing waves«) are marked by triangles in Fig. 2. In these runs the transport rate was large but still largely occurring as bed load. The fact that they all gave values of p close to unity is an experimental support for the idea that p approaches unity for increasing values of θ.

If a limiting value of $p = 1$ is accepted, the expression for p has to be modified as for instance

$$p = \left[1 + \left(\frac{\frac{\pi}{6}\beta}{\theta - \theta_c}\right)^4\right]^{-1/4} \tag{16}$$

which is about equal to Eq. (15) for θ close to θ_c and approaches unity for large values of θ. Eq. (16) is given in Fig. 2 for $\theta_c = 0.05$ and 0.06.

The Suspended Load

For large values of θ' the transfer of shear stress to the bed surface is no longer well described by Eq. (15), because it neglects the dispersive stress from the suspended load. In order to take account of this we have to introduce Bagnold's expression and write

$$\tau' = \tau_c + nF_D + F_s \quad , \tag{17}$$

in which F_s is the dispersive stress as given by Eq. (6), where a specific value of the velocity gradient will have to be inserted. Assuming the classical logarithmic velocity distribution to be at least approximately valid, we get

$$\frac{dU}{dy} = \frac{2.5\,U_f'}{y}$$

The dispersive stress acting on the bed must depend on this velocity gradient calculated for a value of y about equal to one particle diameter d. The following calculations indicated that the value $y = 1.73\,d$ yielded the best agreement with observation of the actual amount of suspension, so that Eq. (17) becomes

$$\tau' = \tau_c + nF_D + 0.027\,\rho s\,(\lambda_b U_f')^2 \quad , \tag{18}$$

where λ_b is the linear concentration at bed level. In nondimensional form this equation becomes

$$\theta' = \theta_c + \frac{\pi}{6}\beta p + 0.027\,s\theta'\,\lambda_b^2 \tag{19}$$

By Eq. (7) we can now calculate the corresponding volumetric bed concentration c_b as

$$c_b = \frac{0.65}{(1 + 1/\lambda_b)^3} \tag{20}$$

Hence this model provides a method for calculation of c_b from the requirements of momentum transfer to the immobile sand surface if p is known.

When θ becomes very large, corresponding to large suspended transport rates, we assume p to be unity and find that

$$\lambda_b = \sqrt{\frac{\theta'-0.3}{0.027\, s\theta'}} \rightarrow 3.74$$

for ordinary sand with $s = 2.65$. This corresponds to the volumetric bed concentration $c_b = 0.32$, which is estimated to be a reasonable maximum value for suspended sediment in motion. Theoretically c_b can be as large as 0.65, but this corresponds to firm packing and does not allow free motion of the particles.

In the general case c_b must be determined from Eqs. (19) and (20), assuming p to be given by Eq. (16) (an illustrative example is given below). For fixed values of θ_c, ß, and s, the bed concentration depends on θ' only. This relationship is presented in Fig. 3 for $\theta_c = 0.05$, $s = 2.65$, and ß = tan 27° = 0.51. Note that c_b becomes extremely small for $\theta' < 0.1$ and that it approaches 0.32 for large values of θ'.

To proceed further it is necessary to obtain an estimate of the size of the particles moving in suspension. This is achieved by means of the criterion

$$w < 0.8\, U_f' \quad , \tag{21}$$

which states that only particles with a fall velocity w smaller than this threshold value will move in suspension. When the distribution curve for w is known it is possible to estimate the »effective fall velocity« for the suspended fraction (Raudkivi 1976). Criteria of the type (21) seem to be generally accepted (Middleton 1976).

When w has been determined the transport rate of suspended load can be calculated from Eqs. (2) and (3), as described by Einstein (1950). The bed load transport is obtained from Eqs. (13) and (16).

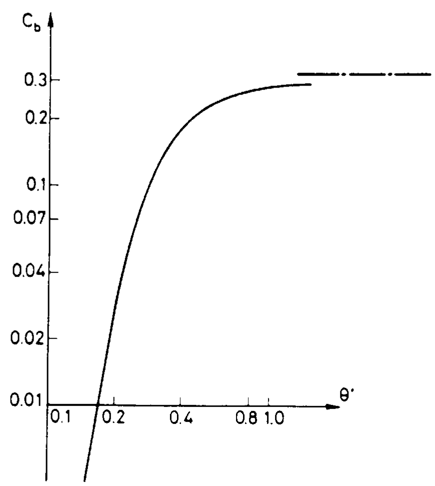

Fig. 3. Bed concentration c_b versus θ', assuming $\theta_c = 0.05$, $s = 2.65$, and ß = 0.51.

The problem is now how the theory can be controlled by comparison with experiments. It is well-known, that measurements of the transport rates of bed load and suspended load separately is difficult and always associated with considerable uncertainty. Likewise, the mean particle size of the suspended load is rather uncertain.

In adapting the Fort Collins data (Guy et al. 1966) we have tried to compare the calculated bed load transport rates with the measurements applying Eq. (14) (which is rather doubtful for large transport rates). Although the scatter is rather large, the general trend in the comparison is satisfactory. A similar remark applies to the suspended particle size.

The total rate of sediment transport, however, can be measured with good accuracy. Hence, the most significant test is to compare the total load with the theory, which is done in Fig. 4.

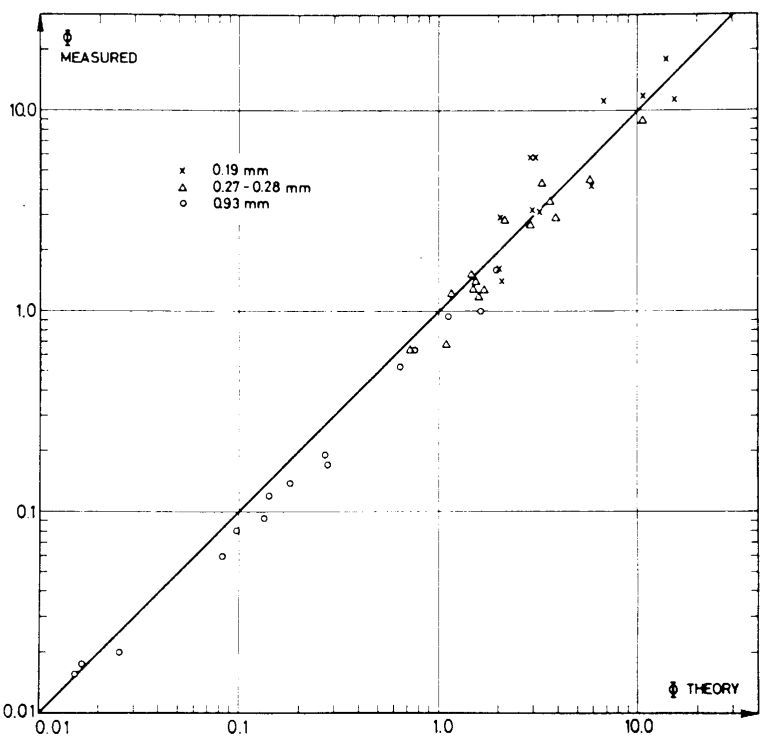

Fig. 4. Comparison between measured and calculated rates at total sediment transport.

Example

As an illustrative example we consider run 21 from the Fort Collins report (Guy et al. 1966) from the series using sand with the mean fall diameter d = 0.28 mm. The data for this run are:

Slope I = 0.00131
Depth D = 0.326 m
Mean velocity V = 0.725 m/s
Temperature T = 16.°5 C

The hydraulic roughness of the surface is estimated to k = 2.5 d. Hence, from Eq. (5) we get D' = 0.116 m and U_f' = 0.0386 m/s. From this we find that

$$\theta' = \frac{D'I}{(s-1)d} = 0.329$$

With θ_c = 0.05 and ß = 0.51 Eq. (16) gives p = 0.859 and the non-dimensional rate of bed load transport is calculated from Eq. (13):

$$\Phi_B = 1.79$$

If the same quantity is estimated from Eq. (14) we get φ_B = 1.22. The critical fall velocity w_c is (Eq. (21)) 0.8 U_f' = 0.031 m/s. From the distribution of particle fall velocity it is found that the mean fall velocity of the suspended part is about w = 0.023 m/s, corresponding to a fall diameter of 0.20 mm. The measured mean diameter of the suspended particles was in this case considerably smaller, about 0.16 mm. (For the runs in general the agreement between measured and calculated particle sizes for the suspended material was better and no trend in the deviations was found).

The value of z becomes

$$Z = \frac{w}{0.4\,U_f'} = 1.49$$

The lower limit a for the integral in Eq. (3) is taken as a = 2 d.

According to Einstein (1950) Eq. (13) can be written as

$$\Phi_s = 11.6\sqrt{\theta'}\,c_b\left(\frac{a}{d}\right)\left[I_1 \ln\frac{30D}{k} + I_2\right] ,$$

where I_1 and I_2 are obtained from his diagrams as 0.40 and -2, respectively. From Eqs. (19) and (20) or from Fig. 3 c_b is found to be 0.14, so that the non-dimensional transport rate of suspended material becomes

$$\Phi_s = 1.94$$

The total sediment transport rate thus becomes

$$\Phi_t = \Phi_b + \Phi_s = 3.73$$

to be compared with the measured value 3.46.

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