MIKE 1D Sediment Transport

Scientific documentation

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CONTENTS

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Scientific Documentation

1   Introduction

This document describes the 1D computational engine for modelling sediment transport and morphological changes.

The sediment transport (ST) module simulates the transport of sediment and the corresponding changes in morphology. The ST module uses the same time step as the HD module and always runs concurrently. The ST module can handle cohesive (mud) and non-cohesive (sand/gravel) components.

Generally, sediment transport can be divided into bed load, suspended load, and wash load. Bed load is the transport of bed material by rolling and sliding along the bed. Suspended load is the material, which is suspended in the fluid part of the time. Wash load is the transport of material continually in suspension and therefore morphologically inactive. In MIKE 1D non-cohesive sediments can behave both like bed load and suspended load. Cohesive sediments behave as suspended load, while wash load can be implemented by setting zero erosion and deposition parameters, or by use of an AD component.

The ST module can be enabled from MIKE HYDRO and from MIKE+. It does not support MIKE 11 ST or MIKE URBAN (classic) systems.

Modelling of the morphological development in rivers and sewers is supported through three different bed layer models. This enables interaction between the bed and the hydrodynamics in both a river and a sewer context.

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The text relevant for MIKE HYDRO can span several paragraphs and stops at the first occurrence of a quadratic black box: ∎

Text that has this graphics tag at its side is only relevant in an urban context, i.e. for people using MIKE+. If MIKE+ is not relevant for you, this text can be omitted.

The text relevant for MIKE+ can span several paragraphs and stops at the first occurrence of a quadratic black box: ∎

2 General overview

2.1 Sediment fractions

MIKE 1D is a multi-fraction sediment transport, morphological and bed layer model. The model operates with multiple sediment fractions with individual characteristics such as grain size, which combined form the local particle size distribution and are transported accounting for effects of the bed surface composition.

MIKE 1D allows both single-fraction and multi-fraction models.

2.2 Type

All sediments can be specified as cohesive or non-cohesive. Cohesive sediments are transported as suspended load (in some cases wash load) while non-cohesive sediments are transported as bed load and/or suspended load, i.e. optionally only one of the transport modes can be selected, as e.g. gravels will usually be bed load only.

The distinguishing factors between the two types of sediments are the grain size, typically non-cohesive fractions for $d_{50} >= 0.1$ mm and cohesive fractions for $d_{50} < 0.1$ mm).

In MIKE+, the "Transport mode" is used to select whether a fraction is cohesive or non-cohesive. Set the transport mode to "ST Model" to get a non-cohesive fraction and "AD Model" to get a cohesive fraction. Non-cohesive fractions will be modelled using bed load only. ∎

2.3 Parameters

Parameters can be divided into overall parameters and parameters specific for the sediment fractions.

The grain size factor d_{90}/d_{50} needs to be specified for single fraction models due to its impact on the skin friction shear stress. If more than one sediment fraction is used, a graded sediment model is used. In this case d_{50} and d_{90} are calculated from the sediment content in the top layer.

For each sediment fraction, the following parameters are needed:

Parameter	Unit	Note
Grain size (d)	m	
Density (s)		Relative density, no unit
Porosity (n)		Common value: 0.3-0.7
Fall velocity (w_s)	m/s	Settling velocity, default is the use of Rubey formula (1933), while a user defined value is also allowed
Critical Shields parameter (θ_c)		Used in some formulas, van Rijn uses its own Critical Shields parameter from the Shields curve, some formulas do not use it.

3 Fall velocity

The fall velocity is by default calculated using the Rubey (1933) formula:

$$w_s = \sqrt{(s-1)gd}\left(\sqrt{\frac{2}{3} + \frac{36v^2}{(s-1)gd^3}} - \sqrt{\frac{36v^2}{(s-1)gd^3}}\right)$$

(3.1)

This formula expresses the fall velocity as a function of the grain size (d), relative density (s) and laminar viscosity (v).

It is also possible to set a user-defined fall velocity, overriding the value calculated by the Rubey formula. Some non-cohesive sediment transport formulas will not allow zero fall velocity due to their mathematical formulations, a nonzero value is the only constraint on the user-defined fall velocity.

Reference

Rubey, W. W. (1933). Settling velocity of gravel, sand, and silt particles. American Journal of Science, (148), 325-338.

4   Einstein decomposition

The total shear stress is decomposed into skin and form for the sediment transport calculations with the following general principles:

- The Rouse number is always calculated from the total friction, being a better reflection of the turbulence levels.

- Interaction with the bed (entrainment, bed load) is reflected by the skin friction unless specifically stated that it is done using the total friction, as in some sediment transport formulas.

Using the Einstein decomposition, the total shear stress is divided into skin and form contributions:

$$\tau = \tau' + \tau'' \qquad (4.1)$$

Where:
τ Total shear stress
τ' Skin friction shear stress
τ'' Form friction shear stress

The same can be used for friction factors:

$$f = f' + f'' \qquad (4.2)$$

Where:
f Friction factor for the total resistance
f' Friction factor for the skin resistance
f'' Friction factor for the form resistance

The relation between shear stress and friction factor is:

$$\tau = \rho f V^2 \qquad (4.3)$$

Where:
ρ Water density
V Flow speed

Here we use the term *friction factor*, while some publications will call *f*/2 the friction factor. The form friction coefficient is also sometimes referred to as a form drag coefficient (e.g. Parker, 2004).

Currently in the MIKE 1D model there are three methods for calculating the skin friction shear stress:

1. Total friction
2. Law of the wall (full depth)
3. Law of the wall (skin friction depth)

If using the first method, the skin friction simply equals the total friction used in the HD model.

Law of the wall (full depth)

The easiest approach for calculating the skin friction is to use the law of the wall, which can be calculated by various means, e.g. from the relation (e.g. van Rijn, 1984):

$$C' = 18 \log_{10}\left(\frac{4h}{d_{90}}\right) \tag{4.4}$$

Where:
C' Chezy number representing skin friction
h Water depth
d_{90} 90% fractile of the local particle size distribution curve

The friction factor for skin resistance can then be calculated from the Chezy number representing skin friction using:

$$f' = \frac{g}{(C')^2} \tag{4.5}$$

Where:
g Gravitational acceleration

Law of the wall (skin friction depth)

The division of the shear stress into skin and form can be extended to the water depths via the static relation for shear stress:

$$\tau = \rho g h S \tag{4.6}$$

Where:
S Water surface slope

Resulting in:

$$h = h' + h'' \tag{4.7}$$

Where:
$\underline{h}$ Total water depth
h' Water depth carried by skin friction
h'' Water depth carried by form friction

Alternative to the simple law of the wall (full depth), the following implicit formulation can be used instead:

$$(f')^{-1/2} = 6 + 2.5\, ln\left(\frac{h'}{3d_{90}}\right) \tag{4.8}$$

We note here that – for legacy reasons – the formulation is slightly different than what is used when the depth is not partitioned. When using this formulation, the expression for f' become implicit with the depth carried by skin friction (h') relating to the total depth by:

$$\frac{h'}{h} = \frac{\tau'}{\tau} = \frac{f'}{f} \tag{4.9}$$

Yielding the implicit equation solved for f':

$$(f')^{-1/2} \; = \; 6 + 2.5 \, ln\left(\frac{f'}{f}\frac{h}{3d_{90}}\right)$$

<div align="right">(4.10)</div>

This equation is solved iteratively where the total resistance is known from the hydrodynamic model. Method 3 with friction-carried depth yields larger skin friction shear stress because the roughness length to depth ratio is seen as larger in the formulation. The difference between Method 2 and Method 3 depends on how much of the total resistance is due to form.

References

Fredsøe, J. & Deigaard, R. (1992) "Mechanics of Coastal Sediment Transport", World Scientific

Parker, G. (2004) "CHAPTER 9: RELATIONS FOR HYDRAULIC RESISTANCE IN RIVERS", downloadable from
http://hydrolab.illinois.edu/people/parkerg/morphodynamics_e-book.htm

5   Ashida & Michiue (1972) hiding function

The Ashida & Michiue (1972) hiding function allows modification of the critical Shields parameter in sediment mixtures:

$$\frac{\theta_c}{\theta_{c,0}} = \begin{cases} 0.85\,\dfrac{d_{50}}{d_i}, & \dfrac{d_i}{d_{50}} < 0.4 \\[2ex] \left(\dfrac{\ln(19)}{\ln\left(19\dfrac{d_i}{d_{50}}\right)}\right)^2, & \dfrac{d_i}{d_{50}} \geq 0.4 \end{cases} \qquad (5.1)$$

Ashida & Michiue is an improvement over the Egiazaroff (1965) hiding function, the latter having a singularity when $d_{50}/d_i=19$.

Ashida & Michiue accounts both for hiding and lubrication by reducing the critical Shields parameter when $d_i>d_{50}$ (exposure) and increasing the critical Shields parameter when $d_i<d_{50}$ (hiding).

Ashida & Michiue can be activated when simulating more than one fraction, but only some formulas react to this activation, namely formulas that use the critical Shields parameter.

In the MIKE HYDRO GUI this is named Egiazaroff. ∎

References

Egiazaroff, I. V. (1965). Calculation of nonuniform sediment concentrations. Journal of the Hydraulics Division, 91(4), 225-247.

Ashida, K., & Michiue, M. (1972). Study on hydraulic resistance and bed-load transport rate in alluvial streams. In Proceedings of the Japan Society of Civil Engineers (Vol. 1972, No. 206, pp. 59-69). Japan Society of Civil Engineers.

6 Surface modification of the sediment transport

Internally the model stores the masses of each fraction in each layer:

$$m_{j,i} = \text{mass of fraction } i \text{ in layer } j \tag{6.1}$$

The so-called surface modification involves the factors F_i, which are calculated as:

$$F_i = f\left(\frac{\Delta_1}{\Delta_a}\right) \frac{m_{1,i}}{\sum m_{1,i}} \tag{6.2}$$

This is the relative mass of the fraction in the bed surface multiplied by the function f, which is given by:

$$f(x) = \begin{cases} x, & x < 1 \\ 1, & x \geq 1 \end{cases} \tag{6.3}$$

The modification for relative mass is the standard surface modification, while the second modification only has an influence if the active layer thickness (Δ_1) is below the target thickness. That situation can only occur under the following conditions:

• 1-layer model with the active layer below its target thickness

• 2-layer model in which the local passive layer is empty and therefore the active layer can go below the target thickness (cannot happen unless the passive layer is empty)

Both situations represent sediment starved conditions, and the modification scales down the sediment transport (f(x)<1).

7   Transport theory

Non-cohesive sediments are transported as bed load and/or suspended load and is always considered in equilibrium. Suspended load is evaluated by the AD solver, using the equilibrium concentration to derive erosion and deposition rates.

Cohesive sediments use the AD solver combined with erosion and deposition.

In MIKE+, non-cohesive sediments are transported using bed load only. ∎

7.1   Bed-load

Bed-load is handled by moving sediment from one grid point to the next in each time step. The amount of sediment is determined from the chosen load formula but limited according to availability of sediment.

In addition to scaling the local bed-load with the weight function F_i, bed-load is also subjected to a supply check, which essentially means that the local bed-load during the considered time-step cannot be larger than what is available in the H-point located just upwind of the Q-point in which the bed-load is calculated. This process ensures mass conservation and stability, but the user should be cautious about the morphological time-step selection, as very large morphological time-steps will not necessarily cause instabilities but will lead to inaccurate solutions. The best way to ensure that a morphological time-step is selected wisely is to run the same model with e.g. half the time-step and compare the results.

MIKE 1D does not include any gravity effects on bed-load, and therefore bed-load and suspended load will give very similar results for relative coarse sediment sizes.

The stability of bed-load is formally limited by the CFL condition, but it may not express itself clearly as instability in layered models due to the supply limitations. However, solutions that do not satisfy the CFL condition are invalid.

7.2   Suspended load (non-cohesive sediment)

The transport of suspended sediment is described by the advection-dispersion equation:

$$\frac{\partial AC_i}{\partial t} + \frac{\partial QC_i}{\partial x} - \frac{\partial}{\partial x}\left(AD \frac{\partial C_i}{\partial x}\right) = W(E_i - D_i) \tag{7.1}$$

Where W is a representative cross-section width. This form applies to both non-cohesive and cohesive sediments, with the erosion and deposition for a non-cohesive sediment fraction:

$$E_i - D_i = \gamma_0 w_s (C_{e,i} - C_i) \tag{7.2}$$

where C is the depth-integrated concentration (of fraction i), w_s is the fall velocity, and C_e is the equilibrium concentration determined from the selected transport formula.

γ_0 is a profile function tabulated in the code as $\gamma_0(Z)$, and it is defined as the ratio between the bed concentration (C_b) of suspended sediment and the depth-integrated concentration (C), in other words the deposition rate is:

$$D_i = \gamma_{0,i} w_{s,i} C_i = w_{s,i} C_{b,i} \tag{7.3}$$

The actual value of γ_0 depends on the distance from the bed (y=b*h) for which the bed concentration applies. The b-value is default b=0.02, but in some formulas, b is specifically set to another value, e.g. Garcia & Parker (1991) uses b=0.05.

7.3 Suspended load (cohesive sediment)

Cohesive sediments are transported in the same manner as non-cohesive suspended sediment, but with different erosion and deposition functions. The erosion rate for cohesive sediment is calculated from:

$$E_i = E_{0,i} \left(\frac{\tau'}{\tau_{ce,i}} - 1 \right)^{n_i}, \quad \tau \geq \tau_{ce,i} \tag{7.4}$$

and deposition is given by:

$$D_i = w_{s,i} C_i \left(1 - \frac{\tau'}{\tau_{cd,i}} \right), \quad \tau \leq \tau_{cd,i} \tag{7.5}$$

The deposition of cohesive sediment is not modified for γ_0.

The following parameters need to be specified for each cohesive fraction:

- Erosion shear stress threshold τ_{ce}
- Erosion coefficient E_0
- Erosion exponent n
- Deposition shear stress threshold τ_{cd}
- Fall velocity w_s

Often one will want to use the same parameters (shear stress thresholds, erosion coefficient and exponent) for all cohesive fractions to represent mass erosion, while the cohesive fractions will obviously have individual fall velocities.

7.4 Sediment transport through nodes

MIKE 1D does not simulate sedimentation in basins and manholes. Both non-cohesive and cohesive sediments continue unaffected through manholes and basins. Suspended load is handled automatically by the AD module, while the bed load entering a node is immediately transferred into outflowreach.

In case the water from the node continues into several reaches (sewer links or river branches), the sediment load is split between the reaches similarly.

7.5 Sediment transport formulas

The following load formulas are available.

- Engelund-Hansen (total load)
- van Rijn (bed and suspended load)
- Meyer-Peter & Müller (total load)

- Yang - Sand (total load)
- Yang - Gravel (total load)
- Engelund-Fredsøe (bed and suspended load)
- Smart-Jaeggi (total load)
- Garcia & Parker (suspended load)
- Wilcock & Crowe (bed load)
- Ackers-White (total load)
- Lane-Kalinske (suspended load)

In MIKE+, not all the transport formulas are available. ∎

Total load formulas determine the total load which is then divided between bed and/or suspended load. The user determines how fractions for bed and suspended load. The total load chosen as suspended load gives the equilibrium concentration:

$$c_e = \frac{q_{susp}\, \rho s}{q_{water}}, \quad q_{water} > 0.$$ (7.6)

If the discharge is zero, the equilibrium concentration is also zero. The internally adopted unit is [g/m³], i.e. the water density is $\rho = 10^6$ g/m³.

The sediment transport formulas are documented in the following sections in which we use the notation explained in the following.

The Shields parameter is defined for each fraction by:

$$\theta_i' = \frac{\tau'}{(s_i - 1)g d_i}$$ (7.7)

Here shown for the skin friction Shields parameter θ', and the Einstein decomposition applies:

$$\theta = \theta' + \theta''$$ (7.8)

We use the standard notation for the Shields parameters:

θ Total Shields parameter
θ' Skin friction Shields parameter
θ'' Form friction Shields parameter

The calculation of these is described in Section 0. The notation also applies to shear stresses, friction velocities, water depths and resistance numbers, e.g. C' means the skin friction Chezy number used in the van Rijn formula.

Calculations involving the suspended sediment concentration profile and turbulence usually are based on the total friction rather than skin friction, while entrainment rates are generally calculated from skin friction. There are no universal laws for how this is to be done, but we follow the principle that everything related to turbulence levels are best derived from total friction, while actions on the bed are best derived from the skin friction, unless the adopted sediment transport model uses the total friction.

Most sediment transport formulas use the non-dimensional sediment transport:

$$q_b = \phi_b \sqrt{(s_i - 1)g d_i^3}$$ (7.9)

Where:
ϕ Non-dimensional sediment transport
q Volumetric sediment transport
g Acceleration of gravity
i Fraction
s Sediment fraction relative density
d Sediment fraction grain diameter

The subscript on the sediment transport formula depends on whether it is bed-load (b), suspended load (s) or total load (t).

For graded sediment transport the sediment transport capacity is multiplied by the F_i weight function, unless explicitly stated that it is not the case.

Some formulas deviate from this, and in those cases it is pointed out in the documentation.

7.5.1 Engelund-Hansen (1967)

Engelund-Hansen (1967) give the formula for the total sediment transport:

$$\phi_t = \frac{0.05}{c_f} \; \theta^{2.5} \tag{7.10}$$

Engelund-Hansen uses the total Shields parameter, and the results hence do not depend on the skin friction calculation method.

For graded sediment the standard approach applies, although the Engelund-Hansen formula was developed for (relatively) uniform sediment.

7.5.2 van Rijn (1984)

van Rijn (1984) published separate formulas for bed-load and suspended load.

The van Rijn model is not a multi-fraction model but is extended in the standard manner to multi-fraction by multiplying the suspended load with the local fraction weight in the active layer. The formula itself is used with the sediment parameters for the individual fractions.

First the so-called transport stage parameter (T) is defined as:

$$T = \frac{\theta'}{\theta_c} - 1 \tag{7.11}$$

The skin friction Shields parameter should be calculated from "Law of the wall, full depth" (set in the GUI) when using van Rijn. When using the van Rijn model the critical Shields parameter given in the GUI is not used. Instead we use the fit to the Shields curve:

$$\theta_c = \begin{cases} 0.24\,D_*^{-1}, & D_* < 4 \\ 0.14\,D_*^{-0.64}, & 4 < D_* < 10 \\ 0.04\,D_*^{-0.1}, & 10 < D_* < 20 \\ 0.013\,D_*^{0.29}, & 20 < D_* < 150 \\ 0.055, & 150 < D_* \end{cases} \tag{7.12}$$

The non-dimensional grain diameter D* is defined by:

$$D_* = d \left(\frac{(s-1)g}{v^2} \right)^{1/3} \tag{7.13}$$

Where v is the kinematic viscosity of the water, which can be set in the GUI. Since the grain size and density are specific for each fraction, the non-dimensional grain size is also specific for each fraction.

The van Rijn (1984) bed-load model yield the simple formula for the non-dimensional bed-load:

$$\phi_b = 0.053 \frac{T^{2.1}}{D_*^{0.3}} \tag{7.14}$$

For suspended load van Rijn (1984) is a bit more complicated. First, we check whether the sediment can go into suspension by checking the criterion:

$$u_f > w_s \max(0.4; \frac{4}{D^*}) \tag{7.15}$$

Here we note that u_f is the total friction velocity, which is more appropriate to use because we are comparing the fall velocity to the vertical turbulent fluctuations.

The bed concentration applies at the distance a from the bed defined by:

$$a = \max(0.01h; 2d) \tag{7.16}$$

And the volumetric bed concentration at the distance a is given by:

$$c_a = 0.015 \frac{d}{a} \frac{T^{1.5}}{(D^*)^{0.3}} \tag{7.17}$$

A special Rouse number Z' is used in the formula (in this case Z' does not mean skin friction), and again we use the total friction velocity to represent the concentration profile:

$$Z' = \frac{w_s}{\beta \kappa u_f} + \varphi \tag{7.18}$$

This is seen to be a special version of the standard Rouse number:

$$Z = \frac{w_s}{\kappa u_f} \tag{7.19}$$

In the modification β is given by:

$$\beta = 1 + 2\left(\frac{w_s}{u_f}\right)^2 \tag{7.20}$$

And the following is added to the Rouse number:

$$\varphi = \frac{5}{2} \left(\frac{w_s}{u_f}\right)^{0.8} \left(\frac{c_a}{c_0}\right)^{0.4} \tag{7.21}$$

Where $c_0 = 0.65$ is maximum possible volumetric concentration.

The volumetric suspended load is calculated from:

$$q_s = F(Z') \, c_a \, V h \tag{7.22}$$

Where h is the water depth and V the depth-integrated flow velocity. The correction factor F(Z') is calculated from:

$$F(Z') = \frac{\left(\frac{a}{h}\right)^{Z'} - \left(\frac{a}{h}\right)^{1.2}}{\left(1 - \frac{a}{h}\right)^{Z'} (1.2 - Z')} \tag{7.23}$$

For graded sediment the fractional loads are modified using the surface weight F_i.

References

van Rijn, Leo C. (1984a) "Sediment transport, part I: bed load transport." Journal of hydraulic engineering 110.10 (1984): 1431-1456

van Rijn, L.C. (1984b) "Sediment Transport Part II: Suspended load transport", Journal of Hydraulic Engineering, Vol. 110, No. 11, November, 1984

7.5.3 Meyer-Peter & Müller (1948)

The Meyer-Peter & Müller formula was originally developed for bed-load, but it is used as a total load formula in MIKE 1D, i.e. the total load can be divided into bed-load and suspended load:

$$\phi_t = 8 \, (\theta' - \theta_c)^{1.5} \tag{7.24}$$

The critical Shields parameter is defined by the user and can be modified by the Ashida & Michiue (1972) hiding function in graded sediment models. The extension to graded sediment is done by the default method by multiplying the loads by the surface weights.

Reference

Meyer-Peter, E., & Müller, R. (1948). Formulas for bed-load transport. In IAHSR 2nd meeting, Stockholm, appendix 2. IAHR.

7.5.4 Yang (1983) – sand

The Yang (1983) formula gives the sediment concentration [ppm]:

$$log_{10} c = A + B \, log_{10} \left(\frac{VI}{w_s} - \frac{V_{cr} I}{w_s} \right) \tag{7.25}$$

The concentration [ppm] is equivalent to [g/m3], see Parker (2004).

Where:
c Concentration [ppm]
V Flow velocity [m/s]
I Water level slope
w_s Settling velocity [m/s]

V_{cr} Critical flow velocity [m/s]

The water level slope is calculated from:

$$I = \frac{u_f^2}{gh}$$ (7.26)

Where:
u_f Friction velocity [m/s] equivalent to total shear stress
h Water depth [m]
gg Acceleration of gravity [m/s2]

The coefficients A and B are functions of the Rouse number and grain Reynolds number:

$$A = 5.435 - 0.286 \, log_{10}\left(\frac{w_s d}{\nu}\right) - 0.457 \, log_{10}\left(\frac{u_f}{w_s}\right)$$ (7.27)

$$B = 1.799 - 0.409 \, log_{10}\left(\frac{w_s d}{\nu}\right) - 0.314 \, log_{10}\left(\frac{u_f}{w_s}\right)$$ (7.28)

The critical flow velocity is determined from:

$$\frac{V_{cr}}{w_s} = \begin{cases} 131, & Re < 1.2 \\ \frac{2.5}{log_{10}Re - 0.06} + 0.66, & 1.2 \leq Re \leq 70 \\ 2.05, & Re > 70 \end{cases}$$ (7.29)

Where Re is a grain Reynolds number:

$$Re = \frac{u_f d}{\nu}$$ (7.30)

The (equilibrium) sediment transport is calculated from the concentration by using:

$$q_t = \frac{qc}{s\rho}$$ (7.31)

Where:
q_t Total sediment transport [m2/s]
q Water flux [m2/s]
s Relative sediment density [-]
ρ Water density [106 g/m3]

The Yang formula is a total load formula, which means the sediment transport has to be divided into bed-load and suspended load:

$$q_b = k_b q_t$$ (7.32)

$$q_s = k_s q_t$$ (7.33)

Where:
q_b Bed-load [m2/s]
q_s Suspended load [m2/s]
Where kb and ks are bed-load and suspended load factors; there is no requirement that the factors sum to unity.

In a graded sediment model the bed-load for each fraction is modified with the relative weight of each fraction in the bed surface (Fi).

In a graded model the following are specific for each fraction (i):

- Grain size d_i
- Relative density s_i
- Settling velocity w_{si}

7.5.5 Yang (1984) – gravel

The Yang (1984) formula for gravel is similar to the formula for sand, but with different coefficients in the A- and B-terms:

$$A = 6.681 - 0.633 \, log_{10} \left(\frac{w_s d}{\nu} \right) - 4.816 \, log_{10} \left(\frac{u_f}{w_s} \right) \tag{7.34}$$

$$B = 2.784 - 0.305 \, log_{10} \left(\frac{w_s d}{\nu} \right) - 0.282 \, log_{10} \left(\frac{u_f}{w_s} \right) \tag{7.35}$$

Reference

Yang, C.T. (1984) "Unit Stream Power Equation for Gravel", Journal of Hydraulic Division, ASCE, 110 (HY12), pp. 1783-1798.

7.5.6 Engelund-Fredsøe

Engelund-Fredsøe refers to two separate formulas for bed-load and suspended load.

Bed-load

Bed-load is calculated from:

$$\phi_b = 5p \left(\sqrt{\theta'} - 0.7\sqrt{\theta_c} \right) \tag{7.36}$$

Where p is interpreted as the probability of sediment movement, and calculated as:

$$p(\theta') = \left(1 + \left(\frac{\frac{\pi}{6} \mu_d}{\theta' - \theta_c} \right)^4 \right)^{-1/4} , \theta' > \theta_c \tag{7.37}$$

The p-value is zero at the critical Shields parameter and increases towards unity for higher Shields parameters.

The extension to graded sediment is standard for Engelund-Fredsøe, and the grain size and porosity are specific for each sediment fraction.

Engelund-Fredsøe uses the GUI specified critical Shields parameter and reacts to Egiazaroff if activated.

When using Engelund-Fredsøe, the skin friction model should be "Law of the wall (skin friction depth)". The Engelund-Fredsøe model was developed for that particular skin friction model.

Suspended load

In the Engelund-Fredsøe model the suspended load is calculated by using a Rouse profile and a logarithmic velocity profile. The bed concentration applies $2d_{50}$ above the bed, which makes the calculations challenging.

The velocity profile is given by:

$$u(y) = \frac{u_f}{\kappa} \log \frac{30y}{k_N} \tag{7.38}$$

The biggest problem with Engelund-Fredsøe is that the lower limit for the concentration profile may be below the lower limit for the velocity profile. In order to avoid that we use the velocity profile defined from the median grain size, i.e. the standard roughness definition:

$$k_N = 2.5d_{50} \tag{7.39}$$

This is not necessarily identical to the roughness used in the skin friction, usually derived from d_{90}. However, the concentration profile starts at $2d_{50}$, so we need to ensure that the velocity profile starts below $2d_{50}$, guaranteed by scaling the Nikuradse roughness with the d_{50} as well.

Note that for single fraction models the use of d_{50} for roughness and lower limit for the concentration profile means that the model will be consistent with the normal use for single fraction. The version developed for MIKE 21C and MIKE 1D is mathematically for multi-fraction models, but it is better to use a model like Garcia & Parker (1991) for multi-fraction, as it is essentially formed on the same theory, but based on multi-fraction data.

Now we can write the velocity profile as ($k_0 = d_{50}/12$):

$$u(y) = \frac{u_f}{\kappa} \log \frac{y}{k_0} \tag{7.40}$$

In order to get rid of the friction velocity, so the profile is solely described by k_0, we calculate the depth-integrated velocity (V):

$$V = \frac{1}{h - k_0} \frac{u_f}{\kappa} \int_{k_0}^{h} \log \frac{y}{k_0} \, dy \tag{7.41}$$

Rearranging:

$$V = \frac{k_0}{h - k_0} \frac{u_f}{\kappa} \left[\frac{h}{k_0} \log \frac{h}{k_0} - \frac{h}{k_0} + 1 \right] \tag{7.42}$$

Rearranging:

$$\frac{u_f}{\kappa} = V \frac{\frac{h}{k_0} - 1}{\frac{h}{k_0} \log \frac{h}{k_0} - \frac{h}{k_0} + 1} \tag{7.43}$$

Inserting into the velocity profile:

$$\frac{u(\eta)}{V} = \frac{\frac{h}{k_0} - 1}{\frac{h}{k_0}\log\frac{h}{k_0} - \frac{h}{k_0} + 1}\left(\log\eta + \log\frac{h}{k_0}\right) \tag{7.44}$$

Engelund-Fredsøe is based on the Rouse profile:

$$c(\eta) = c_b \left(\frac{\eta}{b}\frac{1-b}{1-\eta}\right)^{-Z} \tag{7.45}$$

Where:

$$b = 2\frac{d_{50}}{h} \tag{7.46}$$

Rouse number:

$$Z = \frac{w_s}{\kappa u_f} \tag{7.47}$$

The Zyserman & Fredsøe (1994) bed concentration is applied:

$$c_b(\theta') = \frac{0.331\,(\theta' - \theta_c)^{1.75}}{1 + \frac{0.331\,(\theta' - \theta_c)^{1.75}}{0.46}} \tag{7.48}$$

The suspended load calculated as the integral of the product of the velocity and concentration profiles:

$$q_s\left(\frac{h}{k_0}, Z, \theta'\right) = q\,c_b(\theta')\int_b^1 \frac{\frac{h}{k_0} - 1}{\frac{h}{k_0}\log\frac{h}{k_0} - \frac{h}{k_0} + 1}\left(\log\eta + \log\frac{h}{k_0}\right)\left(\frac{\eta}{b}\frac{1-b}{1-\eta}\right)^{-Z} d\eta \tag{7.49}$$

The integral is now called f(h/k0, Z) and therefore:

$$q_s\left(\frac{h}{k_0}, Z, \theta'\right) = q\,c_b(\theta')\,f(\frac{h}{k_0}, Z) \tag{7.50}$$

This is the suspended load transport capacity, extension to graded sediment is done the usual manner by multiplying with the surface weight F_i. Note that Engelund-Fredsøe is not a true graded sediment model, so it is recommended to use e.g. Garcia & Parker (1991) for graded sediment.

References

Engelund, F. and Fredsøe, J. (1976) "A sediment transport model for straight alluvial channels". Nordic Hydrology, Vol. 7, No. 5.

Zyserman, J. A., & Fredsøe, J. (1994). Data analysis of bed concentration of suspended sediment. Journal of Hydraulic Engineering, 120(9), 1021-1042.

7.5.7 Smart-Jaeggi

The adjustable Smart-Jaeggi formula:

$$\phi = a_1 \left(\frac{d_{90}}{d_{30}}\right)^{a_2} I^{a_3} \left(\frac{C}{\sqrt{g}}\right)^{a_4} \theta^{a_5}(a_6\theta^{a_7} - \theta_c)^{a_8} \tag{7.51}$$

Where:

ϕ	Non-dimensional sediment transport (bed-load or suspended load)
$a_1 - a_8$	Input constants for each fraction and mode
d_{90}/d_{30}	Input parameter for each fraction and mode
I	Bed or surface slope
θ	Shields parameter (total or skin)
θ_c	Critical Shields parameter
C	Chezy number (total or skin friction)
g	Acceleration of gravity 9.81 [m/s^2]

In addition to the listed input parameters (d90/d30 and a1-a8) the following can also be selected:

• Angle of repose
• Skin/total friction
• Bed/surface slope

The Smart-Jaeggi formula is a bed-load formula, but the adjustable formula is allowed for both bed-load and suspended load.

Graded sediment
The term (d90/d30) is not updated in graded models; it is kept at its input value. The reason for this is that Smart-Jaeggi is a single fraction model with (d90/d30) calculated from the gradation of the single fraction.

Sediment transport rates are multiplied by the local weight.

Smart-Jaeggi reacts to Egiazaroff. This means that the critical Shields parameter when using Smart-Jaeggi will reacts to the slope and hiding effects. The basic Smart-Jaeggi formula was developed for single fraction sediment (hence the d90/d30 term), so extensions to graded sediment models happens only via the relative weights.

7.5.8 Garcia & Parker (1991)

Garcia & Parker is a true multi-fraction suspended load model. The advection-dispersion equation form used by Garcia & Parker is different from what we normally use in DHI's models:

$$Erosion - Deposition = w_s(E - c_b) \tag{7.52}$$

This is reformulated from near bed concentration to depth-integrated concentration by using:

$$c_b = \gamma C \tag{7.53}$$

Where γ is the ratio between near bed and depth-integrated concentration, and is calculated from a Rouse profile using b=0.05, i.e. the 5% of the depth used by Garcia & Parker (1991). This results in a function γ(Z), where Z is the Rouse number.

Now the source and sink have the generic form:

$$Erosion - Deposition = w_s(E - \gamma C)$$

(7.54)

Garcia & Parker (1991) provide the entrainment function for sediment mixtures:

$$E_i = F_i \frac{A \, Z_{ui}^5}{1 + \frac{A}{0.3} Z_{ui}^5}$$

(7.55)

Where:
E_i Entrainment rate for the fraction (i)
Z_{ui} Parameter calculated for each fraction (see below)
A Constant, A = 1.3x10⁻⁷
F_i Fraction of (i) in the surface layer, i.e. surface supply modification

E_i is a volumetric concentration, which can be translated to the internally adopted mass concentration units [g/m³] by multiplying by ρs$_i$, , where:
ρ Water density ρ = 10⁶ g/m³
s_i Relative sediment density [-] for the considered fraction

The parameter Z$_{ui}$ is calculated as:

$$Z_{ui} = \lambda_m \frac{u_f'}{w_{si}} Re_{pi}^{0.6} \left(\frac{d_i}{d_{50}}\right)^{0.2}$$

(7.56)

Where:
u_f' Skin friction velocity [m/s]
w_{si} Settling velocity for the considered fraction [m/s]
λ_m Parameter derived from the geometric grain size distribution [-]
Re_{pi} Grain Reynolds number for the fraction
d_i Grain size for the considered fraction [mm]
d_{50} Local median grain size [mm] in the bed surface

The grain Reynolds numbers for the individual fractions are given by:

$$Re_{pi} = \frac{\sqrt{(s_i - 1)g d_i^3}}{\nu}$$

(7.57)

The value of λ$_m$ is given by Garcia and Parker (1991) as:

$$\lambda_m = 1 - 0.288 \, \sigma_\phi$$

(7.58)

Where σ$_φ$ is the standard deviation of the sediment mixture:

$$\sigma_\phi^2 = \sum \frac{m_i}{\sum m_i} (\phi_i - \phi_{mean})^2$$

(7.59)

Where:
m$_i$ Mass of each fraction in the bed surface [g/m²]

with the geometric grain sizes φ_i defined using the phi scale (grain size in [mm]):

$$\phi_i = -log_2 \, d_i \tag{7.60}$$

and the mean geometric grain size (i.e. mean phi size) defined as:

$$\phi_{mean} = \sum \frac{m_i}{\sum m_i} \, \phi_i \tag{7.61}$$

Calculation of the γ-function

The derivation of γ is done from the Vanoni concentration profile:

$$c(y) = c_b \left(\left(\frac{y}{b}\right) \left(\frac{h-b}{h-y}\right) \right)^{-Z} \tag{7.62}$$

Where:
$c(y)$ Concentration profile
y Distance above the bed
h Water depth
b Reference level where b = 0.05h
Z Rouse number [-]

The Rouse number is calculated from the total friction velocity u_f to reflect turbulence:

$$Z = \frac{w_s}{\kappa u_f} \tag{7.63}$$

The function γ=c_b/C is tabulated by numerically integrating the depth-integrated concentration:

$$C = \frac{1}{h-b} \int_b^h c(y) \, dy \tag{7.64}$$

The Vanoni profile now gives a tabulated function:
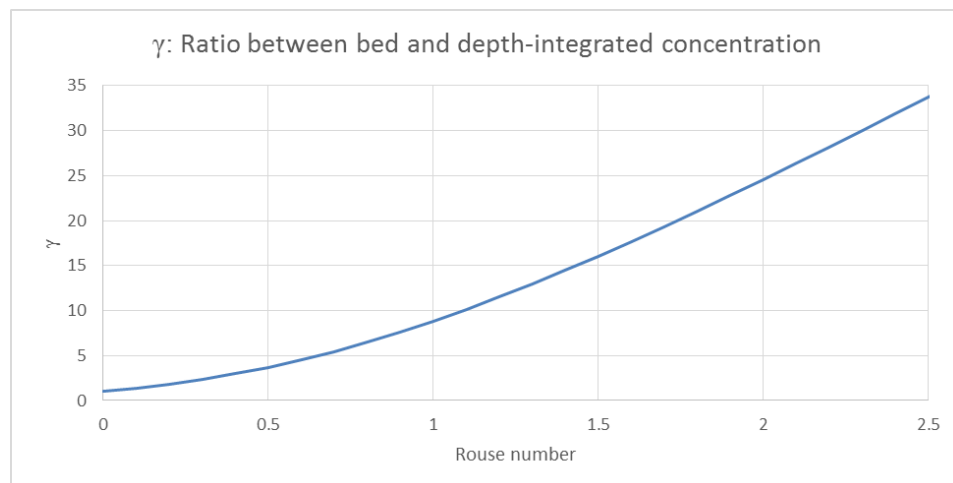
$$\gamma(Z) = \frac{c_b}{C} \tag{7.65}$$

Figure 1   The γ-function

The numerically integrated γ(Z) function is shown graphically above. The parameter is unity for zero Rouse number (i.e. evenly distributed sediment has the same depth-integrated and bed concentrations) and increases quite fast with increasing Rouse number.

Notes regarding Garcia & Parker

Garcia & Parker is a suspended load formula, it does not apply to bed-load.

Garcia & Parker was developed from graded sediment data, and is a true multi-fraction suspended load model.

The model was originally developed for single fraction but was extended to multi-fraction. The single fraction form is imbedded in the presented formulation originally published by the authors.

Garcia & Parker does not use the critical Shields parameter, and therefore does not react to the Ashida & Michiue hiding function, and the formula already includes the surface weight function F_i.

References

Garcia, M., and G. Parker (1991) "Entrainment of bed sediment into suspension", Journal of Hydraulic Engineering, 117(4): 414-435

7.5.9   Wilcock & Crowe (2003)

Wilcock & Crowe (2003) give the following non-dimensional bed-load:

$$W_i^* = \frac{(s_i - 1) g q_{b,i}}{F_i \left(u_f'\right)^3}$$

(7.66)

Where:
W_i^* Non-dimensional bed-load of fraction i
s_i Relative density of fraction i
g Acceleration of gravity
$q_{b,i}$ Volumetric bed-load of fraction i
F_i Relative weight of the fraction (i) in the bed surface

u'_f Skin friction velocity

The non-dimensional bed-load is calculated from:

$$W_i^*(\phi_i) = \begin{cases} 0.002\, \phi_i^{7.5} \, , & \phi < 1.35 \\ 14\left(1 - \dfrac{0.894}{\phi_i^{0.5}}\right)^{4.5} , & \phi \geq 1.35 \end{cases} \tag{7.67}$$

Where φi is given by:

$$\phi_i = \frac{\tau'}{\tau_{ri}} \tag{7.68}$$

Where τ' is the bed shear stress (skin friction) and τri a reference shear stress for fraction i. The reference shear stress for fraction i is given by:

$$\tau_{ri} = \tau_{rg}\left(\frac{d_i}{d_g}\right)^b \tag{7.69}$$

Where:
d_g Geometric mean grain diameter
d_i Grain size for fraction i
τ_{rg} Reference shear stress for the geometric grain size
b Exponent

The exponent b is calculated from:

$$b = \frac{0.67}{1 + \exp\left(1.5 - \dfrac{d_i}{d_g}\right)} \tag{7.70}$$

The reference shear stress for the geometric mean grain size is defined by:

$$\tau_{rg} = \theta_{rg}\left(s_g - 1\right)\rho g d_g \tag{7.71}$$

Where:
s_g Representative relative density for the geometric mean diameter
θ_{rg} Critical shields parameter for the geometric mean grain size

It is noted that the original model did not assume any variation in the s-values for the sediment fractions. In the present implementation we use the average s-value for the non-cohesive sediment fractions. The critical Shields parameter for the geometric mean grain size is calculated from:

$$\theta_{rg} = 0.021 + 0.015 \exp\left(-20 F_s\right) \tag{7.72}$$

Where:
F_s Relative sand content in the bed surface.
F_s is calculated by summing the mass in the active layer of all fractions satisfying the two conditions:

- Fraction is non-cohesive
- Fraction has $d_i < 1$ mm

This is divided by the total mass in the active layer.

Notes regarding Wilcock & Crowe

Wilcock & Crowe is only selectable for bed-load. The Wilcock & Crowe formula was developed for multi-fraction bed-load calculations. The F_i weights are already in the formula, and no other modifications apply.

Wilcock & Crowe can be used also for single fraction bed-load.

Egiazaroff does not apply to Wilcock & Crowe; it uses its own hiding function.

The GUI critical Shields parameter does not apply.

References

Wilcock, P. R., & Crowe, J. C. (2003). Surface-based transport model for mixed-size sediment. Journal of Hydraulic Engineering, 129(2), 120-128.

7.5.10 Ackers & White (1973)

The Ackers & White sediment transport calculation starts with the non-dimensional grain diameter D*:

$$D^* = d \left(\frac{(s-1)g}{v^2} \right)^{1/3} \tag{7.73}$$

Three ranges of D* are considered in the formula:

- $D^* < 1$:   Fine sediment
- $D^* = 1 - 60$:   Transitional
- $D^* > 60$:   Gravel

Four coefficients derive from the non-dimensional grain diameter: n, A, C, m:

$$n = \begin{cases} 1, & D^* < 1 \\ 1 - 0.56 \log_{10}(D*), & 1 \leq D^* \leq 60 \\ 0, & 60 < D^* \end{cases} \tag{7.74}$$

$$A = \begin{cases} 0.37, & D^* < 1 \\ \dfrac{0.23}{\sqrt{D^*}} + 0.14, & 1 \leq D^* \leq 60 \\ 0.17, & 60 < D^* \end{cases} \tag{7.75}$$

$$C = \begin{cases} 2.95x10^{-4}, & D^* < 1 \\ 10^{2.86 \log_{10} D^* - log_{10}^2 D^* - 3.53}, & 1 \leq D^* \leq 60 \\ 0.025, & 60 < D^* \end{cases} \tag{7.76}$$

$$m = \begin{cases} 11, & D^* < 1 \\ \dfrac{9.66}{D^*} + 1.34, & 1 \le D^* \le 60 \\ 1.5, & 60 < D^* \end{cases} \tag{7.77}$$

Ackers & White calculates a general sediment mobility number F_{gr}:

$$F_{gr} = \frac{u_f^n}{\sqrt{(s-1)gd}} \left(\frac{V}{\sqrt{32} \, \log_{10} \dfrac{10h}{d}} \right)^{1-n} \tag{7.78}$$

Where V is the flow speed.

From the general sediment mobility number, the general transport parameter G_{gr}:

$$G_{gr} = C \left(\frac{F_{gr}}{A} - 1 \right)^m \tag{7.79}$$

The general sediment mobility number relates to the sediment transport via:

$$C = G_{gr} \left(\frac{d}{h} \right) \left(\frac{V}{u_f} \right)^n \tag{7.80}$$

Where C is the volumetric sediment concentration. The volumetric sediment transport is hence:

$$q_t = Vh \, G_{gr} \left(\frac{d}{h} \right) \left(\frac{V}{u_f} \right)^n \tag{7.81}$$

Notes regarding Ackers & White

Ackers & White is a total load formula, which can be distributed to bed-load and suspended load by using constant factors. There is no requirement that the formula be used for both bed-load and suspended load, so it is allowed to e.g. use Ackers & White for suspended load and e.g. Meyer-Peter for bed-load.

Ackers & White was developed for single fraction sediment but can be adopted to graded models in which case the load is modified with the local relative weight function (F_i). No other modifications apply to Ackers & White for multi-fraction models.

Reference

Ackers, P., and White, W. R. (1973). "Sediment transport: new approach and analysis," Journal of Hydraulics Division, American Society of Civil Engineers, Vol. 99, No. HY11, pp. 2040-2060.

7.5.11 Lane-Kalinske (1941)

The concentration profile in Lane-Kalinske is calculated from a constant eddy diffusivity instead of the standard parabolic profile. The parabolic profile reads:

$$\nu_T(y) = \kappa u_f y \left(1 - \frac{y}{h} \right) \tag{7.82}$$

Here one will normally use the total friction (u_f) to determine the friction velocity because the total friction better represents the turbulence level compared to skin friction (u_f'). Lane-Kalinske in MIKE 1D is based solely on using total friction in agreement with the original publications.

The depth-averaged eddy viscosity is:

$$\epsilon = \frac{\kappa u_f}{h} \int_0^h y \left(1 - \frac{y}{h}\right) dy = \kappa h u_f \int_0^1 \frac{y}{h} \left(1 - \frac{y}{h}\right) d\left(\frac{y}{h}\right) = \kappa h u_f \int_0^1 s(1-s) ds$$
$$= \frac{\kappa h u_f}{6} = \frac{h u_f}{15}$$

(7.83)

The concentration profile c(y) is found by solving the vertical advection-diffusion equation using this constant diffusivity:

$$w_s c + \epsilon \frac{dc}{dy} = 0$$

(7.84)

With the boundary condition c(a) = c_a. The non-dimensional form is:

$$\frac{w_s h}{\epsilon} c + \frac{dc}{d\eta} = 0$$

(7.85)

Or:

$$\frac{dc}{d\eta} = -\frac{w_s h}{\epsilon} c$$

(7.86)

With the solution:

$$c(\eta) = c_a \exp\left(-\frac{15 w_s}{u_f} (\eta - a)\right)$$

(7.87)

The velocity profile is given by:

$$u(\eta) = 1 + \frac{1}{\kappa} \frac{u_f}{V} (1 + \ln \eta)$$

(7.88)

It is noted that this does not accurately average to unity, and it gives unrealistic profiles for $V/u_f'<5$.

Lane & Kalinske (1941) do not provide the sediment concentration 5% of the water depth; it is given by Lane & Kalinske (1939):

$$C_a = 555 \left(\frac{1}{2} \frac{u_f}{w_s} \exp\left(-\left(\frac{w_s}{u_f}\right)^2\right)\right)^{1.61}$$

(7.89)

Where:
C_a Near-bed concentration [ppm]

Note that [ppm] is approximated by [g/m3].

Originally the equation comes with a factor 5.55 and not 555, but the factor 5.55 is multiplied by a percentage (0-100) of the considered fractions. In MIKE 1D we use

fraction weights (0-1) in the bed surface, and therefore the near-bed concentration should be multiplied by 100 to convert to the use of fraction weights.

It is noted that the skin friction is better for representing the near-bed concentration due to its entrainment nature. However, the work of Lane & Kalinske (1939) operates with turbulent fluctuations, which means the total friction is better than skin friction. Lane & Kalinske did not even mention skin friction in the 1939 or 1941 papers.

Vanoni (1953) notes that the analysis of Lane & Kalinske (1939) was not carried out to a point where the near-bed concentration was associated with some distance from the bed, and therefore the determined value is used as c_a MIKE 1D.

The suspended load capacity is calculated by:

$$q_s = \frac{q}{s\,\rho_w}\, C_0 \left(\frac{w_s}{u_f}\right) P\left(\frac{w_s}{u_f}, \frac{V}{u_f}\right) \tag{7.90}$$

Where:
q Water flux
q_s Suspended load
ρ_w Water density 10^6 g/m^3
ss   Sediment relative density

The P-function is given by:

$$P\left(\frac{w_s}{u_f}, \frac{V}{u_f}\right) = \int_a^1 \left(1 + \frac{1}{\kappa}\frac{u_f}{V}\,(1 + \ln\,\eta)\right) \exp\left(-\frac{15 w_s}{u_f}\,(\eta - a)\right) d\eta \tag{7.91}$$

The P-function is tabulated internally in the code. In the original paper (Lane & Kalinske, 1941), the integral was done from η=0, but this cannot be done because the logarithmic velocity profile is not defined for η=0. Here we choose to integrate from η=a; it is also noted that the adopted logarithmic velocity profile becomes poorly defined for low η-values.

Here we note that the concentration profile is normalized to C(a)=Ca.

The suspended load capacity is translated to an equilibrium sediment concentration.

Notes regarding Lane-Kalinske

This is a suspended load formula, does not apply to bed-load.

Lane-Kalinske can be used in graded models, and indeed Vanoni (1953) gives that the near-bed concentration should simply be scaled with the percentage of the considered fraction in the bed surface. This is equivalent to the standard approach in MIKE 21C and MIKE 1D for multi-fraction models; scaling the near-bed concentration with the relative mass of the fraction in the bed surface is the same as scaling the capacity.

References

Lane, E.W and Kalinske, A.A. (1941) "Engineering calculations of References suspended sediment", Trans.Am.Geophys.Union, vol.22, pp. 603-607.

Lane, E. W., and AAj Kalinske (1939) "The relation of suspended to bed material in rivers." Eos, Transactions American Geophysical Union 20.4 (1939): 637-641.

Vanoni, Vito A. (1953) "A summary of sediment transportation mechanics.", 129-160.

8 Initial conditions

Initial conditions need to be specified for the bed layers and cohesive sediment concentrations. Initial conditions only apply if:

• There is at least one cohesive sediment fraction
And/or
• There is at least one sediment layer

No initial conditions are specified for the initial non-cohesive sediment concentrations, as these are set internally equal to the equilibrium concentrations initially.

Initial cohesive concentrations do not necessarily matter in a model, especially if it starts at low flows and runs long time-scales compared to the influence time-scale of the initial concentration.

The initial conditions for the bed layers are important, and the process involves setting the initial thicknesses of the layers along with particle size distributions for the layers.

There are many tricks that can be used:

• In many cases, one does not have detailed information about the bed layers.

• It is possible in a 2-layer model to only specify a thickness of one of the layers along with a particle size distribution; the bed layer model will automatically balance the active and passive layer thicknesses during the first time-step.

• The depositional problems the initial bed composition may not be important.

9 Morphological updating

The morphological changes during a simulation are calculated separately for each sediment fraction in the form of local mass changes in each cross-section. Those changes are calculated by separate means for bed-load and suspended load.

Bed-load

For the bed-load, the mass changes are calculated using the Exner equation, i.e.:

$$\frac{\Delta m_{1,i}}{\Delta t} = \rho s_i \left(Q_{b,in,i} - Q_{b,out,i} \right) \tag{9.1}$$

Where $m_{1,i}$ [g/m^2] denotes the sediment mass in the active layer of the considered fraction (i) per surface area in the cross-section, and $Q_{b,i}$ are the volumetric sediment transport rates [m^3/s] in and out of the cross-section.

A very simple numerical scheme is used for the calculation, in which the time-derivative is simply forward in time and the advective term can be calculated directly upstream and downstream of each cross-section due to the staggered grid adopted in MIKE 1D. In order to ensure a stable solution, the bed-load calculations adopt upwind flow areas when calculating the flow velocities used for the shear stress. This is done to prevent wiggles in the solution.

Suspended load

For the suspended load, we cannot use the Exner equation due to the accumulation of sediment in the water column (especially for fine sediment). Instead we use the source/sink terms in the advection-dispersion equation directly:

$$\frac{\Delta m_{1,i}}{\Delta t} = D_i - E_i \tag{9.2}$$

The (for each fraction) erosion (E_i) and deposition (D_i) denote the fluxes of sediment between the water and bed per surface area per time [g/m^2/s].

9.1 Layer model

The layer model has three different modes.

0-layer

This choice is only available when simulating one single non-cohesive sediment component and corresponds to unlimited availability of sediment.

1-layer

Limited availability of sediment. The initial layer thickness is given directly as input or initialized as the difference between the initial bottom level and non-scouring bed level. A 1-layer model is always numerically valid, but physically it works best in situations with very thin sediment layers transported over fixed surfaces, e.g. sediment transport in pipes.

2-layer

Limited availability of sediment. Active and passive layer with interaction between the layers. A 2-layer model requires at least two sediment fractions in order to be selectable.

When using a 2-layer model, an extra step enters into the calculation loop. The step is performed after updating the passive layer (layer-1), which will deviate from the active layer thickness (target thickness) when it is updated. The two layers are now updated in an attempt to keep layer-1 equal to the target thickness. The thicknesses equivalent to the masses in the bed layers (j=1-2) are first calculated:

$$\Delta_j = \sum_i \frac{m_{j,i}}{s_i * \rho * (1 - n_i)}$$
(9.3)

The change in the interface between layer-1 and layer-2 can now be calculated:

$$\Delta z = \Delta_1 - \Delta_a$$
(9.4)

Where:
Δz Change in interface elevation
Δ_1 Thickness of layer-1
Δ_a Active layer thickness (model parameter)

The highest theoretical value for Δz is Δ_1, which happens if the active layer thickness is zero. Although not possible to set the active layer thickness to zero, it can be set very low. In this case all the sediment in layer-1 should move to layer-2.

The minimum value of Δz is $-\Delta_a$, which occurs when layer-1 is depleted (empty) during the time-step. In this case the interface should move down with a level change equal to the active layer thickness to get layer-1 to a thickness equal to the active layer thickness. However, this is not always possible because the thickness of layer-2 needs to be taken into consideration as well, such that the interface cannot move down more than the thickness of layer-2, i.e. combined with the upper limit we find:

$$-\Delta_2 \leq \Delta z \leq \Delta_1$$
(9.5)

What this states is simply that the interface cannot move further down than the bottom of layer-2.

Having now established the interface change between layer-1 and layer-2, it is time to calculate the associated sediment exchange.

First consider the case where Δz is positive. In this case Δz will move from layer-1 to layer-2 with the bed composition of layer-1. This we can write as:

$$m_{1,i} \leftarrow m_{1,i} - m_{1,i} \frac{\Delta z}{\Delta_1} = m_{1,i} \left(1 - \frac{\Delta z}{\Delta_1}\right)$$
(9.6)

$$m_{2,i} \leftarrow m_{2,i} + m_{1,i} \frac{\Delta z}{\Delta_1}$$
(9.7)

When looking at this we see that the sediment mass in layer-1 is scaled down with the same relative reduction for all sediment fractions, i.e. the relative distribution in layer-1 does not change, while the relative fraction composition in layer-2 will change.

The case where the interface moves down (Δz negative) means that the fraction $\Delta z / \Delta_2$ of layer-2 will move to layer-1, i.e.:

$$m_{1,i} \leftarrow m_{1,i} - m_{2,i} \frac{\Delta z}{\Delta_2} \tag{9.8}$$

$$m_{2,i} \leftarrow m_{2,i} + m_{2,i} \frac{\Delta z}{\Delta_2} = m_{2,i} \left(1 + \frac{\Delta z}{\Delta_2} \right) \tag{9.9}$$

Here the situation is reversed, such that the layer-2 masses are scaled with the same factor, i.e. no change in the layer-2 composition, while the layer-1 composition changes because layer-1 receives sediment from layer-2.

One notices that the range for Δz ensures in a very logical way here that each layer cannot be more than emptied.

9.2   Non-scouring bed level

Level below which there is no available sediment. Bottom level cannot come below this level.

Where both an initial layer depth and a non-scouring bed level is defined, the bottom level for the sediments is the largest of the two, i.e. the non-scouring bed level will only have an effect if it protrudes the initial layer depth level.

9.3   Cross section updating

The cross sections can be updated with three different methods.

MODEL 1   MODEL 4   MODEL 5
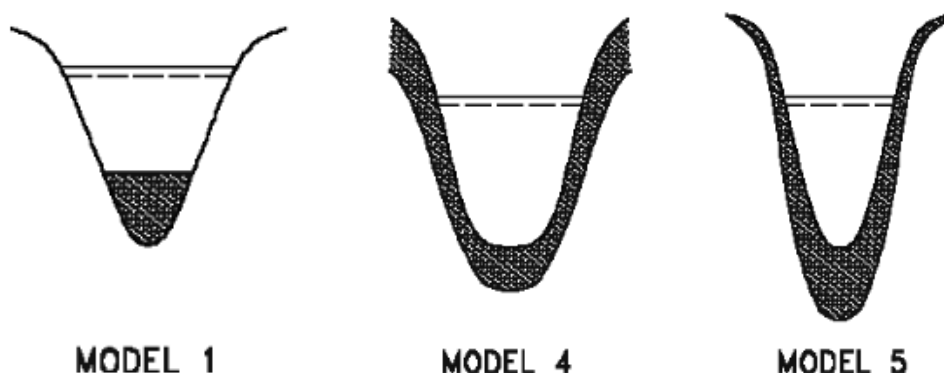
Model 1
Sediment settles with a horizontal surface independently of cross section beneath sediment level. This method is usually used for pipes and other cross sections with a fixed, hard bottom level, typically made of concrete or plastic.

Model 4
Sediment settles equally over the entire cross sections. As a result, the cross section is not changed but simply moved up and down when the sediment erodes or deposits.

Model 5
Sediments settles more in bottom of cross section. The deposition is scaled with the depth of the cross section below the flood plain level. The flood plain level is by default

the top of the cross section but can be user specified or set to be the smallest of the low flow bank markers. When cross section is almost fully filled up to the flood plain level, Model 4 is applied for any further deposition.

10 Open Boundary conditions

Three different open inflow boundary conditions are available.

10.1 Sediment transport

Sediment inflow is specified for each sediment component. If relevant, the inflow is divided into bed load and suspended load by the engine such that the total inflow is equal to the specified inflow.

10.2 Bottom level

One can choose either to specify the bottom level at relevant boundaries or specify the bottom level change. Both can be given as constants or time series.

10.3 Sediment supply

Sediment supply assumes the inflow to match the sediment load capacity in the first grid point. It has the same effect as setting bottom level change of zero.

11 Passive branches

It is possible to specify branches as passive if they are not relevant or do not contribute to the sediment simulation. Passive branches will not transport any sediment and there is no morphological updating.

This is useful for tributaries or link channels where it is known that sediments do not pass.

12 Sediment removal

Basins and structures can act as "sediment traps" because the water flow kinetic energy and turbulence (hence, dynamic forces acting on sediments) in the basin or before the structure are much lower than in the river branches or sewer links.

Only the bed load is affected by sediment removal, and hence only non-cohesive sediment fractions are affected.

The sediment removed can be reported in the ST output file as removed sediment load or total removed sediment mass (instantaneous or accumulated) for each fraction separately and accumulated for all fractions.

12.1 Sediment removal in nodes

MIKE 1D does not simulate sedimentation in basins and manholes. However, effect of basins on sediment transport (non-cohesive bed load only) may be emulated by defining a sediment removal efficiency in individual basins. This function removes the specified fraction of the inflowing non-cohesive (ST transport) sediments. The efficiency is defined as:

$$E = 1 - \left(1 + \frac{1}{n}\frac{w}{Q/A}\right)^{-n} \tag{12.1}$$

Where:
E sediment trap efficiency
w fall velocity
Q discharge into the node
A Surface area of node
n turbulence constant indicating the settling performance of the basin

    $n = 1$, for poor performance,
    $n = 3$, for good performance,
    $n > 5$, for very good performance,
    $n = \infty$, for ideal performance.

The removed sediments are taken out of the computations. No morphological modelling takes place of the removed (deposited) sediment and it is not possible to re-suspend this sediment.

12.2 Sediment removal in structures

Structures such as weirs can act as efficient barriers for bed load transport, as the weir crest level can be significantly higher that the reach invert level. This means that significant part of non-cohesive sediment cannot be transported over weirs. This is typically applied in sewer context.

MIKE 1D simulates effect of weirs on bed load transport by defining a constant sediment removal efficiency factor or removal efficiency factor as function of weir discharge.

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